Midterm Exam - November 24, 2016, Stochastic Analysis

Family name ________________________ **Given name** ________________________

Signature ________________________ **Neptun Code** ________________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (5 marks) Calculate the variance of $\int_0^3 e^{2B_t-2t}\mathrm{d}B_t$.

   Hint: The moment generating function of $X \sim \mathcal{N}(\mu,\sigma^2)$ is $M(\lambda) = e^{\mu\lambda+\frac{1}{2}\sigma^2\lambda^2}$.

2. (5 marks) Give an explicit formula for the cumulative distribution function $F(x) = \mathbb{P}(X \le x),\ x \in \mathbb{R}$ of the random variable X, where
$$X = \int_0^2 \frac{\mathrm{d}B_s}{1+B_s^2} - \int_0^2 \frac{B_s\mathrm{d}s}{(B_s^2+1)^2}.$$

   Hint: You may use $\Phi(\cdot)$, the c.d.f. of the standard normal distribution in your solution.

3. (5 marks) Let (B_t) and $(\widetilde{B}_t)$ denote i.i.d. standard Brownian motions. Show that $M_t = 4B_t^3\widetilde{B}_t - 4B_t\widetilde{B}_t^3$ is a martingale by writing it as $M_t = \int_0^t X_s\mathrm{d}B_s + \int_0^t \widetilde{X}_s\mathrm{d}\widetilde{B}_s$ for some adapted processes (X_t) and $(\widetilde{X}_t)$.