**Second make-up midterm - December 7, 2016, 17.15-18.00, Stochastic Analysis**

**Family name** ______________________ **Given name** ______________________

**Signature** ______________________ **Neptun Code** ______________________

**No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.**

1. Let us define $g(x) = 2x + 1$. We approximate the stochastic integral $\mathcal{I} = \int_0^1 g(B(t))\mathrm{d}B(t)$ with the sum

$$\mathcal{I}_n = \sum_{k=1}^{n} g\left(B\left(\frac{k-1}{n}\right)\right) \cdot \left(B\left(\frac{k}{n}\right) - B\left(\frac{k-1}{n}\right)\right).$$

   (a) (1 marks) Find the simple predictable function $(X_n(t))$, $0 \le t \le 1$ for which $\mathcal{I}_n = \int_0^1 X_n(t)\mathrm{d}B(t)$.

   (b) (4 marks) What is the smallest value of $n$ for which $\mathbb{E}\left((\mathcal{I} - \mathcal{I}_n)^2\right) \le \frac{1}{100}$ holds?

2. (5 marks) Find the Doob-Meyer decomposition $X_t = A_t + M_t$ of

$$X_t = e^{-4B_t - 8t},$$

   where $A_t$ is an adapted process with finite total variation and $M_t$ is a martingale. Write $M_t$ as an Itō integral w.r.t. Brownian motion.

3. (5 marks) Let $(X_t)$ and $(Y_t)$ denote Itô processes that satisfy

$$X_t = 1 - \int_0^t Y_s \mathrm{d}B_s, \quad Y_t = \int_0^t X_s \mathrm{d}B_s.$$

   Find the exact value of $Z_t = X_t^2 + Y_t^2$ for any $t \ge 0$. *Hint:* First calculate $\mathrm{d}Z_t$.