Second make-up midterm - December 13, 2016, 10.15-11.00, Stochastic Analysis

Family name ____________________________ **Given name** ____________________________

Signature ____________________________ **Neptun Code** ____________________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. Let us define
$$X_t = e^{-2t} X_0 + 2 \int_0^t e^{2(u-t)}\, \mathrm{d}B_u,$$
where X_0 is independent of (B_t) and $X_0 \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = -1$ and $\sigma^2 = 9$.
   (a) (1 marks) Calculate $\mathbb{E}(X_t)$ for any $t \geq 0$.
   (b) (4 marks) Calculate $\mathrm{Var}(X_t)$ for any $t \geq 0$.
2. Let us define $Y_t = \int_0^t \left(t^2 - s^2\right) \mathrm{d}B_s$ for any $t \geq 0$.
   (a) (3 marks) Show that (Y_t) is an *Itô process* by rewriting it in the form $Y_t = Y_0 + \int_0^t \mu_s \mathrm{d}s + \int_0^t \sigma_s \mathrm{d}B_s$.
   (b) (2 marks) Calculate the quadratic variation $[Y]_t$.
3. (5 marks) Find a non-negative process (Z_t) satisfying
$$\mathrm{d}Z_t = Z_t \mathrm{d}B_t + Z_t \mathrm{d}t, \qquad Z_0 = 2.$$
Hint: First calculate the stochastic differential of $\log(Z_t)$ using Itô's formula for Itô processes.