Second make-up midterm - December 15, 2016, Stochastic Analysis

Family name ____________________ **Given name** ____________________

Signature ____________________ **Neptun Code** ____________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is allowed.

1. (5 marks) Find the covariance of $\int_0^2 (2B_s - 1)\,\mathrm{d}B_s$ and $\int_0^3 (B_s^2 + 1)\,\mathrm{d}B_s$.
2. (a) (3 marks) Use Itō calculus to show that

$$M_2(t) = B_t^2 - t, \qquad M_4(t) = B_t^4 - 6tB_t^2 + 3t^2$$

   are martingales. *Hint:* First calculate the stochastic differential of $(M_2(t))$ and $(M_4(t))$.

   (b) (2 marks) Find the adapted process $(\sigma_t)_{0\le t\le 1}$ for which $B_1^4 = \mathbb{E}[B_1^4] + \int_0^1 \sigma_t\,\mathrm{d}B_t$.

   Hint: First find the process $(\widetilde{\sigma}_t)_{0\le t\le 1}$ for which $B_1^2 = \mathbb{E}[B_1^2] + \int_0^1 \widetilde{\sigma}_t\,\mathrm{d}B_t$.
3. (5 marks) Find a non-negative process (Z_t) satisfying

$$\mathrm{d}Z_t = -Z_t\mathrm{d}B_t + Z_t\mathrm{d}t, \qquad Z_0 = 3.$$

   Hint: First calculate the stochastic differential of $\log(Z_t)$ using Itô's formula for Itô processes.