**Midterm Exam - May 25, 2023, Stochastic Analysis**

**Family name** ____________________ **Given name** ____________________

**Signature** ____________________ **Neptun Code** ____________________

**No calculators or electronic devices are allowed. One formula sheet with 15 formulas is OK.**

1. (7 points) Let $(B_t)$ denote standard Brownian motion and $(\mathcal{F}_t)$ its natural filtration. Let

$$X = (B_4)^3.$$

   (a) Calculate the conditional expectation $M_t := \mathbb{E}(X \mid \mathcal{F}_t)$ for all $0 \le t \le 4$.

   (b) Find the adapted process $(\sigma_t)_{0 \le t \le 4}$ for which $X = \mathbb{E}[X] + \int_0^4 \sigma_s \, \mathrm{d}B_s$.

2. (8 points) Let us consider the Itô process that satisfies

$$X_t = 5 + 3\int_0^t X_s \, \mathrm{d}s + 2\int_0^t X_s \, \mathrm{d}B_s, \qquad t \ge 0.$$

   (a) Find the value of $x \in \mathbb{R}$ for which $\mathbb{P}(X_4 \le x) = \frac{1}{2}$.

   (b) Let $Y_t = \sqrt{X_t}$. Show that $(Y_t)$ is a time-homogeneous Itô diffusion process by writing down the drift coefficient $\mu : \mathbb{R} \to \mathbb{R}$ and the diffusion coefficient $\sigma : \mathbb{R} \to \mathbb{R}$ for which $\mathrm{d}Y_t = \mu(Y_t)\mathrm{d}t + \sigma(Y_t)\mathrm{d}B_t$.