Make-up Midterm Exam - May 31, 2023, Stochastic Analysis

Family name ____________________ **Given name** ____________________

Signature ____________________ **Neptun Code** ____________________

No calculators or electronic devices are allowed. One formula sheet with 15 formulas is OK.

1. (7 points) The Itō process (Y_t) satisfies

$$Y_t = \frac{2}{3} - \int_0^t \frac{Y_s}{1+s}\,\mathrm{d}s + 60 \cdot \int_0^t \frac{1}{1+s}\,\mathrm{d}B_s, \qquad t \geq 0.$$

   Find the cumulative distribution function of Y_1.

   Instruction: Express the result using $\Phi(\cdot)$, the c.d.f. of standard normal distribution.

2. (8 points) The Itō process (Z_t) satisfies

$$\mathrm{d}Z_t = 8\,\mathrm{d}t + \sqrt{8Z_t}\,\mathrm{d}B_t, \qquad t \geq 0, \qquad Z_0 = 60.$$

   For any $x \in \mathbb{R}$, let $T_x := \min\{\, t \geq 0 \,:\, Z_t = x \,\}$. Calculate $\mathbb{E}(T_{120})$.

   Hint: Find a function $g : \mathbb{R}_+ \to \mathbb{R}$ such that $M_t := g(Z_t) - t$ is a martingale.

   Instruction: You may assume without proof that $\mathbb{P}(T_0 < +\infty) = 0$. You may use the optional stopping theorem without checking its conditions.