1.5

$\binom{12}{5}$ WAYS TO CHOOSE 5 MEN OUT OF 12

$\binom{10}{5}$ WAYS TO CHOOSE 5 WOMEN OUT OF 12

ONCE THE 5 MEN AND 5 WOMEN ARE PICKED, THERE ARE 5! WAYS TO PAIR THEM.

SO $\binom{12}{5} \cdot \binom{10}{5} \cdot 5!$ RESULTS ARE POSSIBLE.

1.8

a) IMAGINE 20 BALLS IN A ROW YOU ALSO INSERT 3 STICKS BETWEEN SOME OF THE BALLS LIKE THIS:

$$\underbrace{0000}_{\text{FOUR CHOCO}}\,|\underbrace{\,}_{\text{ZERO STRAWBERRY}}|\,\underbrace{00\ldots0}_{\text{7 WALNUT}}\,|\,\underbrace{00\ldots0}_{\text{9 COCONUT}}$$

$\binom{23}{3}$ BALL-STICK ARRANGEMENTS $\Rightarrow$ $\binom{23}{3}$ CAKE TRAY ARRANGEMENTS

$\boxed{1.8}$ b) 16 BALLS, 3 STICKS

$\binom{19}{3}$ BALL - STICK ARRANGEMENTS

BUT NOW IF BALL ARRANGEMENT IS

$$\underbrace{0\,0}_{\boxed{2}}\,|\underbrace{\,}_{\boxed{0}}|\,\underbrace{0\ldots0}_{\boxed{8}}\,|\,\underbrace{0\ldots0}_{\boxed{6}}$$

THEN

ADD ONE TO EACH:

$2+1=3$ CHOCO $\qquad$ $8+1=9$ WALNUT

$0+1=1$ STRAWB. $\qquad$ $6+1=7$ COCONUT

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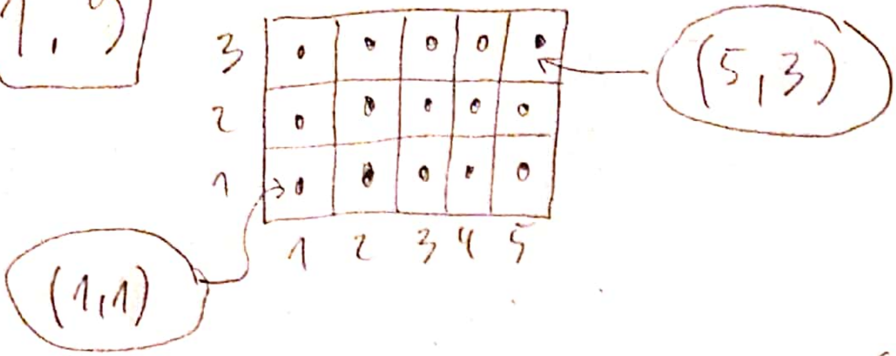
c) I ORDER EITHER ONE OR TWO CAKES FROM EACH KIND, SO THERE ARE $2\cdot2\cdot2\cdot2=2^4$ WAYS

(TWO POSSIBILITIES FOR EACH KIND OF CAKE)

1.9

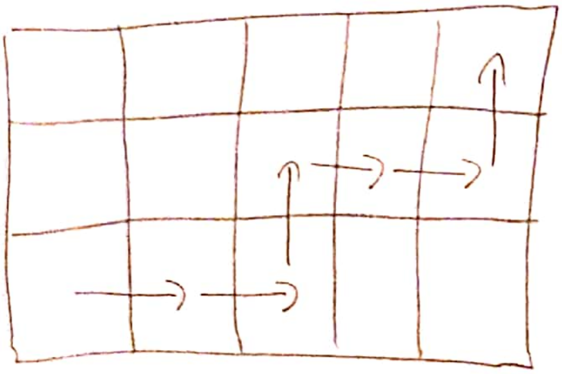


ENCODE EACH PATH AS A SEQUENCE OF $\boxed{R}$ AND $\boxed{U}$ SYMBOLS

$\boxed{R}$ ↑ RIGHT

$\boxed{U}$ ↑ UP

E.G.:



TRANSLATES TO:

$\boxed{RRURRU}$

EACH CODE HAS 4 $\boxed{R}$ SYMBOLS AND 2 $\boxed{U}$ SYMBOLS.

NUMBER OF CODES = NUMBER OF PATHS =

$$\binom{4+2}{2} = \binom{6}{2} = \frac{6!}{2!\cdot 4!} = \frac{6\cdot 5}{2!} = 15$$

$\boxed{1.14}$ $\binom{18}{4}$ WAYS TO PICK FOUR ELKS OUT OF 18.

$\binom{5}{2}$ WAYS TO PICK 2 ELKS OUT OF 5 MARKED.

$\binom{13}{2}$ WAY TO PICK 2 ELKS OUT OF 13 UNMARKED.

$\binom{13}{2} \cdot \binom{5}{2}$ WAYS TO PICK 4 ELKS OUT OF 18 SUCH THAT 2 ARE MARKED, 2 ARE NOT MARKED

IF A IS THE EVENT THAT EXACTLY TWO ARE MARKED THEN

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{\binom{13}{2} \cdot \binom{5}{2}}{\binom{18}{4}}$$

$\boxed{1.17}$ H = HEADS. T = TAILS

$$\Omega = \left\{ \begin{matrix} \boxed{HH}, & \boxed{HTT}, & \boxed{HTHH}, & \boxed{HTHTT}, & \ldots \\ \boxed{TT}, & \boxed{THH}, & \boxed{THTT}, & \boxed{THTHH}, & \ldots \end{matrix} \right\}$$

$\mathbb{P}(HTT) = \frac{1}{8}$ BECAUSE THERE ARE 8 HEAD-TAIL SEQUENCES OF LENGTH 3, EACH SEQUENCE IS EQUALLY LIKELY.

SIMILARLY: $\mathbb{P}(HTHH) = \frac{1}{16}$

$$\mathbb{P}(A) = \sum_{n=2}^{5} \Big(\underbrace{2^{-n}}_{\text{STARTS WITH } \boxed{H}} + \underbrace{2^{-n}}_{\text{STARTS WITH } \boxed{T}} \Big) = 2 \cdot \left(2^{-2} + 2^{-3} + \ldots + 2^{-5} \right) =$$

$$= \sum_{r=1}^{4} \left(\frac{1}{2}\right)^{r} = \frac{1}{2} \cdot \sum_{r=0}^{3} \left(\frac{1}{2}\right)^{r} = \frac{1}{2} \cdot \frac{\left(\frac{1}{2}\right)^{4} - 1}{\frac{1}{2} - 1} = \frac{15}{16}$$

$$\mathbb{P}(B) = \sum_{n \text{ EVEN}} \left(2^{-n} + 2^{-n} \right) = 2 \cdot \left(2^{-2} + 2^{-4} + 2^{-6} + \ldots \right) =$$

$$= 2^{-1} + 2^{-3} + 2^{-5} + \ldots = \frac{1}{2} \cdot \sum_{r=0}^{\infty} \left(\frac{1}{4}\right)^{r} = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{4}} = \frac{2}{3}$$

$\boxed{1.3}$ a) $8! = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

b) FIRST, MERGE A AND B INTO ONE PERSON
NOW THERE ARE 7! SEATING
ARRANGEMENTS. THERE ARE TWO
WAYS TO UNMERGE EACH:
$\boxed{AB}$ OR $\boxed{BA}$
THUS THE ANSWER IS: $2\cdot 7!$

c) $2\cdot 4!\cdot 4!$ SINCE THERE ARE 4!
WAYS TO ARRANGE THE MEN, 4!
—"— —"— —"— WOMEN AND
2 WAYS TO START AN ALTERNATING
SEATING OF THEM
(EITHER START WITH A MAN
OR —"— —"— A WOMAN)

d) LIKE IN b): $4!\cdot5!$
(MERGE AND UNMERGE MEN)

e) $4!\cdot 2^4$ (MERGE COUPLES, 4! ARRANGEMENTS,
EACH COUPLE CAN BE UNMERGED
IN TWO DIFFERENT WAYS)

$\boxed{1.12}$ $|\Omega| = n^n$ (EACH OF THE n BALLS HAVE n CHOICES)

$A = \{$ EXACTLY ONE URN IS EMPTY $\}$

$= \{$ ONE EMPTY, ONE WITH TWO BALLS $\}$

THUS $|A| = \binom{n}{2} \cdot n \cdot (n-1)!$

SINCE THERE ARE $\binom{n}{2}$ WAYS TO CHOOSE WHICH TWO BALLS GO TOGETHER, MERGE THEM.

THERE ARE n WAYS TO CHOOSE THE EMPTY LOCATION.

NOW WE HAVE $n-1$ OBJECTS AND $n-1$ LOCATIONS, ONE GOES IN EACH: $(n-1)!$

$$\mathbb{P}(A) = \frac{|A|}{|\Omega|} = \frac{\binom{n}{2} \cdot n \cdot (n-1)!}{n^n}$$