Probability 1 - Practice

Week 2 12th Sep 2024

2.1 Let A, B and C be three events. Use set theory operations to express the following events:

(a) Out of A, B and C, at least k events occur ($k = 1, 2$).

(b) Out of A, B and C, exactly k events occur ($k = 0, 1, 2$).

2.2 How many different outcomes can the experiment have? Identify the sample space of the experiment:

(a) We throw 3 different coins and 2 identical dice.

(b) We throw 3 identical black dice, and 2 identical white dice.

2.3 What has a higher probability? Throwing a die four times, and rolling 6 at least once, or throwing two dice 24 times, and rolling 6 on both together at least once.

2.4 From a population of n balls in a bucket, we color R to be red, and the rest of them to be blue. Now, let's choose a sample of size N from the population. What is the probability that none of the selected balls are red if

(a) we put them back into the bucket after each draw? (drawing with replacement)

(b) we don't put them back into the bucket after each draw? (drawing without replacement)

(c) Still drawing without replacement. What is the probability of choosing exactly i red balls, for $0 \leq i \leq \min\{N, R\}$ fixed?

2.5 There are 37 pockets on the roulette wheel: 18 black, 18 red and 1 green. We spin the wheel 13 times. What is the probability of rolling each color at least once?

2.6 Santa Claus has 40 dark chocolate bars and 20 white chocolate bars. He distributes the chocolate bars randomly among ten children in a way that each child gets six chocolate bars. What is the probability that each of the ten children get at least one white chocolate bar?

HW 2.7 (2 points) We use a 52 card French card deck to play bridge. There are 4 players (N,E,S,W) and each get exactly 13 cards. What is the probability that N has exactly k aces? (k = 0,1,2,3,4)

♣ **2.8** (3 points) Show that for any A, B events:

$$-\frac{1}{4} \leq \mathbb{P}(A)\mathbb{P}(B) - \mathbb{P}(A \cap B) \leq \frac{1}{4}$$

2.9 (a) Let A and B be two events. If $\mathbb{P}(A) \geq 0.8$ and $\mathbb{P}(B) \geq 0.5$ then prove the following:

$$\mathbb{P}(A \cap B) \geq 0.3$$

(b) Let's prove the following equality for any $A_1, A_2, \ldots, A_n$ events:

$$\mathbb{P}(A_1 \cap A_2 \cap \cdots \cap A_n) \geq \mathbb{P}(A_1) + \mathbb{P}(A_2) + \cdots + \mathbb{P}(A_n) - (n-1)$$

HW 2.10 (3 points) (a) We throw a die 6 times. What is the probability, that $1, 2, 3, 4, 5$ and 6 all get thrown?

(b) We throw a dice 11 times. What is the probability that $1, 2, 3, 4, 5$ and 6 all get thrown at least once?

2.11 An urn contains 6 red, 4 blue, and 5 green balls. What is the sample space, if a set of 3 balls is randomly selected without replacement?

(a) What is the probability that each of the balls will be of the same color?

(b) What is the probability that each of the balls will be of different colors?

HW 2.12 (2 points) There are 7 red, 7 white, and 8 blue balls in an urn. We draw 6 without replacement. What is the probability that we draw at least one of each color?

2.13 We are watching a horse race with 7 horses. Let A be the event that $HorseA$ is either first, second or third, while B is the event that $HorseB$ is second. What is the probability of $A \cup B$?

2.14 We deal all cards from a well shuffled French deck (52 cards). What is the probability that

(a) the ace of hearts is the 11th distributed card?

(b) the first distributed ace is the 11th distributed card?

(c) the first 4 cards are of different suits? ($\clubsuit, \spadesuit, \heartsuit, \diamondsuit$)

(d) an ace never follows the king of the same suit directly? (that is eg. $K\clubsuit A\clubsuit$ do not appear in the sequence)

HW 2.15 (3 points) 12 couples sit down randomly to a round table. What is the probability that no husband sits next to his wife?

2.16 In a certain village, there are 20 families: 5 families with a single child, 7 families with 2, 4 families with 3, 3 families with 4 and 1 family with 5 children.

(a) We choose a family at random. What is the probability that there are i children in this family?

(b) We choose a child at random. What is the probability that the family of this child has i children?