$\boxed{2.5}$ $B := \{$ WE ROLL BLACK AT LEAST ONCE $\}$

$R := \{$ — 〃 — RED — 〃 — $\}$

$G := \{$ — 〃 — GREEN — 〃 — $\}$

$$\mathbb{P}(B \cap R \cap G) \overset{\boxed{\text{DE MORGAN}}}{=} 1 - \mathbb{P}(B^c \cup R^c \cup G^c)$$

$$\mathbb{P}(B^c \cup R^c \cup G^c) \overset{\boxed{\text{INCLUSION-EXCLUSION}}}{=}$$

$$= \mathbb{P}(B^c) + \mathbb{P}(R^c) + \mathbb{P}(G^c) - \mathbb{P}(B^c \cap R^c) +$$

$$- \mathbb{P}(B^c \cap G^c) - \mathbb{P}(R^c \cap G^c) + \mathbb{P}(B^c \cap R^c \cap G^c)$$

$$\mathbb{P}(B^c) = \mathbb{P}(\text{WE NEVER ROLL BLACK}) = \left(1 - \frac{18}{37}\right)^{13}$$

$$\mathbb{P}(R^c) = \left(1 - \frac{18}{37}\right)^{13} \quad \mathbb{P}(G^c) = \left(1 - \frac{1}{37}\right)^{13}$$

$$\mathbb{P}(B^c \cap R^c) = \mathbb{P}(\text{WE NEVER ROLL BLACK AND RED})$$
$$= \mathbb{P}(\text{WE ONLY ROLL GREEN}) = \left(\frac{1}{37}\right)^{13}$$

$$\mathbb{P}(B^c \cap G^c) = \left(\frac{18}{37}\right)^{13}, \quad \mathbb{P}(R^c \cap G^c) = \left(\frac{18}{37}\right)^{13}$$

$$\mathbb{P}(B^c \cap R^c \cap G^c) = \mathbb{P}(\emptyset) = 0$$

$\boxed{2.6}$ $A_i = \{$CHILD i GETS AT LEAST ONE WHITE CHOC. BAR$\}$

$$\mathbb{P}\left(\bigcap_{i=1}^{10} A_i\right) \overset{\boxed{\text{DE MORGAN}}}{=\!=\!=} 1 - \mathbb{P}\left(\bigcup_{i=1}^{10} A_i^c\right)$$

$$\mathbb{P}\left(\bigcup_{i=1}^{10} A_i^c\right) \overset{\boxed{\text{INCLUSION-EXCLUSION}}}{=} \sum_{\substack{I \subseteq [10] \\ I \neq \emptyset}} (-1)^{|I|+1} \cdot \mathbb{P}\left(\bigcap_{i \in I} A_i^c\right) = \star$$

$$\mathbb{P}\left(\bigcap_{i \in I} A_i^c\right) = \mathbb{P}\left(\forall i \in I : \text{CHILD } i \text{ GETS NO WHITE}\right) =$$

$$= \frac{\binom{60-6\cdot|I|}{20}}{\binom{60}{20}}$$

$\boxed{\text{AND IF } 60-6\cdot|I| < 20 \text{ THEN THE NUMERATOR IS ZERO.}}$

$$\star = \sum_{r=1}^{10} \sum_{\substack{I \subseteq [10] \\ |I| = r}} (-1)^{r+1} \cdot \frac{\binom{60-6\cdot r}{20}}{\binom{60}{20}} =$$

$$= \sum_{r=1}^{10} \binom{10}{r} \cdot (-1)^{r+1} \cdot \frac{\binom{60-6\cdot r}{20}}{\binom{60}{20}} =$$

$$= \binom{10}{1} \cdot \frac{\binom{54}{20}}{\binom{60}{20}} - \binom{10}{2} \cdot \frac{\binom{48}{20}}{\binom{60}{20}} + \binom{10}{3} \cdot \frac{\binom{42}{20}}{\binom{60}{20}} - \dots$$

$\boxed{2.11}$ $6+4+5=15$ DISTINGUISHABLE BALLS

Ω: IS THE SET OF 3-ELEMENT SUBSETS OF THE ABOVE 15-ELEMENT SET

a) $\mathbb{P}$(3 BALLS HAVE THE SAME COLOR) =

$$= \underbrace{\mathbb{P}(3\text{ RED})}_{\frac{\binom{6}{3}}{\binom{15}{3}}} + \underbrace{\mathbb{P}(3\text{ BLUE})}_{\frac{\binom{4}{3}}{\binom{15}{3}}} + \underbrace{\mathbb{P}(3\text{ GREEN})}_{\frac{\binom{5}{3}}{\binom{15}{3}}}$$

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b) $\mathbb{P}$(3 BALLS HAVE DIFFERENT COLOR) =

$\mathbb{P}$(ONE RED, ONE BLUE, ONE GREEN) =

$$= \frac{6 \cdot 4 \cdot 5}{\binom{15}{3}}$$

$\boxed{2.13}$ HORSE A = ADMIRAL

HORSE B = BRAVEHEART

„OCCAM'S RAZOR" $\Longrightarrow$ ALL 7! HORSE SUCCESSIONS ARE EQUALLY LIKELY

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

$$\mathbb{P}(A) = \frac{3 \cdot 6!}{7!} = \frac{3}{7}$$

$$\mathbb{P}(B) = \frac{6!}{7!} = \frac{1}{7}$$

$$\mathbb{P}(A \cap B) = \mathbb{P}(B \cap A) = \frac{2 \cdot 5!}{7!} = \frac{1}{7} \cdot \frac{2}{6}$$

ALTERNATIVELY:

$$\mathbb{P}(B \cap A) = \underbrace{\mathbb{P}(B)}_{\frac{1}{7}} \cdot \underbrace{\mathbb{P}(A|B)}_{\frac{2}{6}}, \text{ WHERE}$$

$$\boxed{\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}}$$

$\mathbb{P}(A|B)$ = THE CONDITIONAL PROBABILITY OF A GIVEN THAT B OCCURS

$\boxed{2.14}$ a) $\frac{1}{52}$

b) $\frac{\binom{52-11}{3}}{\binom{52}{4}} = \frac{\binom{41}{3}}{\binom{52}{4}}$ BECAUSE WE ALREADY KNOW THAT POSITIONS $1, 2, \dots, 10$ HAVE NO ACE, POSITION 11 HAS AN ACE, SO 3 ACES HAVE TOBE DISTRIBUTED ON 52-11 POSITIONS

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c) $\frac{4! \cdot (13)^4 \cdot (48)!}{52!} = \frac{4! \cdot 13^4}{52 \cdot 51 \cdot 50 \cdot 49} = \frac{3! \cdot 13^3}{51 \cdot 50 \cdot 49}$

ALTERNATIVE SOLUTION:

$A_i = \{$ THE FIRST i CARDS HAVE DIFFERENT SUITS $\}$

$A_2 \supseteq A_3 \supseteq A_4$

$$P(A_4) = \underbrace{P(A_2)}_{\frac{3 \cdot 13}{51}} \cdot \underbrace{P(A_3 | A_2)}_{\frac{2 \cdot 13}{50}} \cdot \underbrace{P(A_4 | A_3)}_{\frac{13}{49}}$$

THE PRODUCT RULE OF CONDITIONAL PROBABILITY

$\boxed{2.14}$ d)

$A_i :=$ { ACE OF SUIT i FOLLOWS KING OF SUIT i }

$i = 1, 2, 3, 4$

$\boxed{\text{DE MORGAN}}$

$$\mathbb{P}(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c) = 1 - \mathbb{P}\left(\bigcup_{i=1}^{4} A_i\right)$$

$\boxed{\text{INCL.-EXCL.}}$

$$\mathbb{P}\left(\bigcup_{i=1}^{4} A_i\right) = \sum_{\substack{I \subseteq [4] \\ I \neq \emptyset}} (-1)^{|I|+1} \cdot \mathbb{P}\left(\bigcap_{i \in I} A_i\right)$$

$$\mathbb{P}\left(\bigcap_{i \in I} A_i\right) = \frac{(52-|I|)!}{52!}$$ (MERGE KING i AND ACE i $\forall i \in I$)

$$\text{☆} = \sum_{r=1}^{4} \sum_{\substack{I \subseteq [4] \\ |I| = r}} (-1)^{r+1} \cdot \frac{(52-r)!}{52!} = \sum_{r=1}^{4} \binom{4}{r} \cdot (-1)^{r+1} \cdot \frac{(52-r)!}{52!} =$$

$$= \binom{4}{1} \cdot \frac{1}{52} - \binom{4}{2} \cdot \frac{1}{52 \cdot 51} + \binom{4}{3} \cdot \frac{1}{52 \cdot 51 \cdot 50} + \binom{4}{4} \cdot \frac{1}{52 \cdot 51 \cdot 50 \cdot 49}$$

$\boxed{2.3}$

A = { THROW DIE 4 TIMES, GET 6 AT LEAST ONCE }

B = { THROW 2 DICE 24 TIMES, GET 6 ON BOTH AT LEAST ONCE }

$$\mathbb{P}(A) = 1-\left(1-\frac{1}{6}\right)^4$$

$$\mathbb{P}(B) = 1-\left(1-\frac{1}{36}\right)^{24}$$

$$\boxed{\mathbb{P}(A) \overset{?}{>} \mathbb{P}(B)} \iff \boxed{1-\left(1-\frac{1}{6}\right)^4 \overset{?}{>} 1-\left(1-\frac{1}{36}\right)^{24}}$$

$$\iff \boxed{\left(1-\frac{1}{36}\right)^{24} \overset{?}{>} \left(1-\frac{1}{6}\right)^4} \iff \boxed{\left(1-\frac{1}{36}\right)^{6} \overset{?}{>} 1-\frac{1}{6}}$$

BERNOULLI'S INEQUALITY:

$(1+x)^r > 1+r\cdot x$ IF $r = 2,3,4,\dots$ AND $x \geq -1$, $x \neq 0$

NOW CHOOSE $\boxed{r=6}$ AND $\boxed{x = \frac{-1}{36}}$ TO OBTAIN $\left(1-\frac{1}{36}\right)^6 > 1-\frac{1}{6}$, THUS $\mathbb{P}(A) > \mathbb{P}(B)$

$\boxed{2.16}$

a) $A_i := \{$ i CHILDREN IN THIS FAMILY $\}$

$$\mathbb{P}(A_1) = \frac{5}{20}, \quad \mathbb{P}(A_2) = \frac{7}{20}, \ldots, \mathbb{P}(A_5) = \frac{1}{20}$$

b) $A_i^* := \{$ i CHILDREN IN THIS FAMILY $\}$

$$5 \cdot 1 + 7 \cdot 2 + 4 \cdot 3 + 3 \cdot 4 + 1 \cdot 5 = 48$$

$$\mathbb{P}^*(A_1^*) = \frac{5 \cdot 1}{48}, \quad \mathbb{P}^*(A_2^*) = \frac{7 \cdot 2}{48}, \ldots \mathbb{P}^*(A_5^*) = \frac{1 \cdot 5}{48}$$

„SIZE-BIASED SAMPLING"