Probability 1 - Practice

Tutorial no. 3   19th Sep 2024

3.1 We put N balls into n boxes, with all n^N possibilities having equal probability. Assuming that a given box is not empty, what is the conditional probability that there are K balls in that box $(1 \leq K \leq N)$?

3.2 We throw three dice. Assuming that the number on each die is different, what is the conditional probability that at least one of them is a 6?

3.3 Die α has 4 red and 2 white sides, while die β has 2 red and 4 white sides. We toss a coin. If the outcome is heads, we use die α, and if the outcome is tails, we use die β. We throw n times with the die chosen.

(a) What is the probability, that the k-th throw lands on red? $(k = 1, 2 \ldots, n)$

(b) Assuming that the first throw was red, what is the conditional probability that the second throw will also be red? $(k = 1, 2 \ldots, n)$

(c) Assuming that the first $k - 1$ throws were red, what is the conditional probability that the k-th throw will also be red? $(k = 1, 2 \ldots, n)$

3.4 There are 2 balls in an urn. They were painted to either black or gold with probability $\frac{1}{2} - \frac{1}{2}$ independently.

(a) We have been told that at least one of the balls is gold. What is the probability of both balls being gold?

(b) We pull out a ball and it is golden. What is the probability that the other ball is also golden?

HW 3.5 (3 points) We throw with one blue, one yellow, one green and one red die. Let the three numbers thrown be B, Y, G and R.

(a) What is the probability that all four numbers thrown are different?

(b) Assuming that all four numbers are different, what is the conditional probability of $B < Y < G < R$?

(c) How much is $\mathbb{P}(B < Y < G < R)$?

3.6 Three chefs, A, B and C are trying to bake special cakes. They have 0.02, 0.03 and 0.05 probabilities of overcooking a cake, respectively. A bakes 50% of the cakes, B bakes 30% and C bakes the remaining 20% of cakes. What percent of the ruined cakes did A bake?

3.7 0.1% of human fetuses have the Down syndrome. There is a clinical test that for each fetus with the Down syndrome gives a false negative with probability 1%, while for each fetus without the syndrome gives a false positive with probability 5%. The test tells a couple that their fetus has Down syndrome. What is the probability that this is indeed the case?

♣ **3.8** (3 points) A hunter sees a fox at a distance of 30 meters and shoots at it. If the fox survives, it runs away with a speed of 10 m/s. It takes 3 seconds for the hunter to reload and shoot again, continuously. The fox has a $\frac{1}{4}$ probability of surviving a shot, no matter how many shots have hit him previously. We know that:

$$\mathbb{P}(\text{The hunter hits the fox from a distance of x meters}) = \frac{675}{x^2}$$

Does the fox have a positive probability of surviving?

HW 3.9 (3 points) Shane spends $\frac{3}{4}$ of his day at the pub. There are four bars in the village, and because he has no preference, he picks each pub with an equal probability and spends the day there. We try to find him and visit three bars, but he wasn't in any of them. What is the conditional probability that he is in the fourth pub?

3.10 (Monty Hall problem) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

HW 3.11 (2+2 points) There are two urns, with red and blue balls. In the first urn, there are 4 red and 5 blue balls, while in the second urn, there are 3 red and 7 blue balls.

(a) Scenario 1: we take a ball from the first urn and put it into the second urn. We then take a ball from the second urn and put it into the first urn. Then, we take another ball from the first urn. What is the probability that this ball is red?

(b) Scenario 2: we take 1-1 ball from the two urns at the same time, and switch them. Then we take a ball from the first urn. What is the probability that this ball is red?

3.12 A private facebook group can only be joined if someone from the group invites you. Originally, the group has two members: Adam and Eve. Occasionally, a random person from the group invites a new member. Someone is part of Adam's "clique", if they are Adam themselves, or if someone from Adam's clique invited them.

(a) What is the probability of Adam's clique having k members when the facebook group has 4 members? $(k = 1, 2, 3)$

(b) Let X be the number of people in Eve's clique when the facebook group has n members. What is the probability of the event $\{X = k\}$ for $k \in \{1, \ldots, n-1\}$?. *Hint:* Use mathematical induction.

3.13 100 passengers are waiting to board an airplane with 100 seats. The first person to board the airplane lost their ticket, and so they sit down randomly in one of the seats. The following people board the plane individually, and try to sit down in the seat written on their ticket. If they see someone already sitting in their seat, they sit down randomly in one of the empty seats. What's the probability that the last person to board the plane ends up sitting in the correct seat?