Probability 1 – Exercises

Tutorial no. 5 — 3rd Oct 2024

5.1 Assuming that the ratio of left-handed people is 1% on average, what is the probability that out of 200 randomly selected people, at least 4 are left-handed?

HW 5.2 (2 points) On average, how many raisins need to be in a muffin, if we want to make sure that for a randomly selected muffin, the probability of it containing at least 1 raisin is at least 98%?

5.3 What is the expected value and the variance of a die roll?

5.4 We're reading Glamour Magazine, and we notice that there are around 0.2 typos each page, on average. What is the probability that the next page has (a) 0 typos (b) 2 or more typos?

5.5 At a horse race, the horses run laps and there are many obstacles that they have to jump over. My horse, "Lucky Star", has an equally small probability to knock each obstacle down, independently of the other obstacles, and the probability that she does a lap perfectly is 0.1.

(a) What is the probability that she knocks down at least 4 obstacles in one lap?

(b) At each mistake she gets an injury with probability 1/2, independently of what happens at any other obstacle. What is the probability that she finishes a lap without any injuries?

5.6 A Marathon is held next to the Danube Bend. Sadly, the route was led through a section of tall grass, which was full of ticks. We later found out, that 300 people found one tick on their skin, and 75 found two. Based on this, approximate how many people ran in the race, overall.

HW 5.7 (3 points) In Oblivia, the rate of suicide is 1 per $100,000$ people every month. Let's examine a city in Oblivia with $400,000$ people.

(a) What is the probability that at least 3 people will commit suicide in a given month?

(b) What is the probability that in a given year, there will be at least 1 month with at least 3 suicides?

(c) Starting now, what is the probability that the first month with at least 3 suicides will be i months from now? ($i \geq 1$)

5.8 We are rolling a die until getting the second 5.

(a) What is the expected number of rolls we need?

(b) What is the probability that we need more rolls than this expectation?

5.9 We have 10 red, 20 blue and 40 green balls in an urn. We randomly select 7. What is the distribution of the number of blue balls selected? What is their expected number?

5.10 We throw 10 times with a fair coin. Let X be the number of sequences with the same result (For example if $HHTTTTHTHH$ then $X = 5$). What is the distribution of X?

5.11 Let $\mathbb{E}(X) = 1$ and $\mathbb{D}^2(X) = 5$. Calculate the following:

(a) $\mathbb{E}((2+X)^2)$

(b) $(\mathbb{E}(2+X))^2$

(c) $\mathbb{D}^2(4+3X)$

5.12 We have two dice, one red and one green.

(a) What is the expected value of the minimum of the two numbers, and the maximum of the two numbers?

(b) We threw with both dice 3 times. I only looked at the red one, but I saw that it was 3 each time. What is the probability, that it was bigger than the green one at least once?

HW 5.13 (2 points) We throw three times with a biased coin which gives heads with probability 2/3. Let U be the number of times we throw the previous result again (so U can be $0, 1$ or 2. For example, if HHH then $U = 2$, if HTH then $U = 0$). Calculate the expected value and variance of U!

HW 5.14 (3 points) We throw 10 times with a biased coin that has $\frac{7}{10}$ probability of heads. After each throw that lands on tails, we have to pay $300Ft$, however, we get $200Ft$ after each throw that lands on heads. Calculate the expected value and variance of our winnings at the end of the game.

5.15 Let N be a non-negative, integer valued random variable with $\mathbb{E}(N^3) < \infty$.

Show that:

$$\sum_{i=1}^{\infty} i^2 \mathbb{P}(N \geq i) = \frac{\mathbb{E}(N^3)}{3} + \frac{\mathbb{E}(N^2)}{2} + \frac{\mathbb{E}(N)}{6}$$

♣ **5.16** (3 points) Two people with the same abilities are running a race, on an infinitely long track. Both of their lanes have the same obstacles (ditches), and they both have 2^{-L} probability to jump over a ditch, where L is the width of the ditch, independently of each other or the previous ditches. If one of them falls in a ditch, then the race is over. If both of them fall in the same ditch, then it is a tie, and we start over.

(a) Show that the probability of a tie is less than ε exactly if $L < \log_2(\frac{1+\varepsilon}{1-\varepsilon})$

(b) How big should L be, if we want to minimize the expected number of jumps until someone wins?

5.17 In the center of London, the number of traffic accidents follows a Poisson distribution with parameter 20 on rainy days and parameter 10 on sunny days. In early Novermber in London, the probability that it rains (the whole day) is 0.6; the probability that the weather is clear is thus 0.4. According to The Times, 17 car accidents happened in London's center last Thursday. What is the probability that it was a rainy day?