# Probability 1 - Exercises

Tutorial no. 6 — 10th Oct 2024

**6.1** Show that if $X \sim \text{POI}(\lambda)$ and $Y \sim \text{POI}(\mu)$ and if $X$ and $Y$ are independent and if $Z = X + Y$ then $Z \sim \text{POI}(\lambda + \mu)$.

**HW 6.2** (2 points) Leo, when he goes hiking, has a small, equal and independent probability to fall with each step and hit his knee, or fall and hit his elbow. On a 10 kilometer hike, on average, he hits his knee 2 times, and his elbow three times. At most, how long of a hike can his mother let him go on if we want a $\frac{3}{4}$ probability for him to not hit his knee or elbow?

**6.3** The police frequently checks the speed of cars on the highway ("radaring"). Experience shows that if they turn on the radar for five minutes then the probability of detecting at least one speeding car is the same as the probability of not detecting any.

(a) What is the probability that after 20 minutes of radaring, (i) exactly 2, (ii) at least 2 speeding cars will be caught?

(b) How long should the police radar for if they want to catch at least 1 speeding car with a 95% probability?

**6.4** Let's assume that the number of gold ores in a given area has $Poi(10)$ distribution. Each gold ore, independently of the others, is discovered with probability $\frac{1}{50}$. Calculate the probability that

(a) exactly 1,

(b) at least 1, and

(c) at most 1 ore is discovered this time.

Hint: First show that if a random variable has $Poi(\lambda)$ distribution, and each event is counted with $p$ probability, independently of each other, then the number of events counted is $Poi(\lambda \cdot p)$.

**HW 6.5** (2 points) The average density of trees in a forest is 15 trees in $100m^2$. The trunk of the trees are perfect cylinders, with a diameter of $30cm$ each. We shoot a gun $130m$ away from the edge of the forest, without aiming in the direction of the edge of the forest. What is the probability that we hit a tree? (Note: Disregard the fact that the centre of two trees can't be closer than 30cm to each other).

**6.6** Ulysses wants to go home to Ithaca, but he angered Poseidon, who put the ocean full of whirlpools. The density of whirlpools is 2 whirlpools in every 10 square kilometers. If a boat gets closer than $150m$ to a whirlpool, it sinks. Otherwise, the ship remains unharmed. Odysseus doesn't know where the whirlpools are, he just goes straight to Ithaca. He studied Probability 1 at BME, so he knows that he has a 1% probability to make it home safely. How far away is Odysseus from Ithaca?

**6.7** Milka introduced a new type of chocolate bar that contains raisins and peanuts. Each chocolate bar consists of 15 squares. Each chocolate bar has 30 raisins in it on average, and the probability that a square of chocolate has no peanuts is $\frac{1}{2}$.

(a) What is the distribution of the peanuts in the chocolate bar?

(b) We break off a piece that consists of two squares. What is the probability that the combined number of peanuts and raisins in this piece of chocolate is less than 2?

**6.8** We throw with a fair die until each number from 1 to 6 has occurs at least once. What is the expected value of the number of throws that we need for this?

**HW 6.9** (2 points) There are 4 red and 4 blue balls in an urn. We pick 4 balls at random out of the urn. If we picked 2 red and 2 blue, we stop, otherwise we put them back. We continue until we manage to pull out exactly 2 red and 2 blue. What is the probability that we pull exactly $n$ times?

**6.10** Amy and Bernadette play the following game. Amy throws with a fair die until the second occurrence of the number 6. She does not tell Bernadette which throw she managed to throw 6 for the first time, but she does tells her which throw she managed to land on 6 for the second time. After this, Bernadette needs to guess when the first 6 could have occurred. What should she guess? If she is smart, with what probability that she guesses correctly?

♣ **6.11** (3 points) Let $X$ be a Poisson distribution with parameter $\lambda$. Show that

$$\mathbb{P}(X \text{is even}) = \frac{1}{2}(1 + \exp(-2\lambda)))$$

**6.12** Andrew and Bella are playing the following game. They throw 2 fair dice, and Andrew pays Bella the same amount of forint as the square of the difference of the two numbers, while Bella pays Andrew the sum of the two numbers. Who is favoured in this game?

**HW 6.13** (1+2+1 points) Two people are competing in archery. The contestants have a $p_1$ and $p_2$ probability to hit the target ($p_1 < p_2$) with each shot. The person who is less skilled starts, and then they alternate. The first person to hit the target wins.

(a) What is the probability that the more skilled person wins?

(b) What is the expected time of the game, if there is one shot every minute?

(c) Give a simple expression for the expected time if $p_1 = p_2$. Check that the formula in part b gives the same result when $p_1 = p_2$.

**6.14** (2 points) The municipality of Randomburg provides free wifi for the citizens. Each wifi hotspot has $50m$ range, and they placed 3 hotsposts per square kilometer randomly. At the very moment when Bob left his house, the university sent an email to all the students to notify them that all teaching activity is cancelled for today due to a bomb alarm. Bob goes to university on a straight path, and he has 70% chance to receive the notification via the free wifi on the way. How far away is Bob's house from the university?