Probability 1 – Exercises

Tutorial no. 8 24th Oct 2024

8.1 Most IQ tests follow a Normal Distribution with a mean of 100 points and a standard deviation of 15 points. If we believe that these tests are accurate, then

(a) What percentage of people have an IQ between 95 and 110?

(b) How big of an interval around 100 points does 50% of the population fall into?

(c) In a town with 2500 people, on average, how many people will have at least 125 IQ?

HW 8.2 (2 points) Let's say we still believe that these IQ tests (see the previous exercise) are accurate. The test results are divided into three categories; low, medium and high IQ. Out of the participants, 20%, 65% and 15% fall into these categories, respectively. Where should we draw the boundaries between these categories?

8.3 Approximate the probability that out of 100000 poker hands dealt, the number of full houses is between 128 and 158. (In a poker hand of 5 cards, a full house consists of 3 cards of one height, and 2 cards of another height.)

8.4 In Budapest, p percent of people smoke, but we don't know how many. We want to guess their number, so we ask n people, and if k people say they smoke, we guess that p is approximately $p' = \frac{k}{n}$. How many people should we ask if we want to be closer than 0.02 to the truth with probability at least 93%, no matter what the truth is? In other words, how big should we choose n if we want

$$\mathbb{P}(|p' - p| \leq 0.02) \geq 0.93.$$

HW 8.5 (2 points) A factory produces two kinds of coins: fair ones and biased ones; a biased one lands on heads with probability 58%. We have a coin from this factory, but do not know which kind. To decide, we perform the following statistical test: we throw the coin 1000 times, and if it lands on heads at least 540 times, we decide it is biased; if it lands on heads less than 540, we consider it to be fair. What is the probability that we are wrong in the case when the coin is fair? And if the coin is biased?

8.6 The time between two metros in Budapest follows an Exponential distribution with an expected time of 2 minutes. I just missed a metro:

(a) What's the probability, that I have to wait at least 5 minutes for the next metro?

(b) I've been waiting for 3 minutes. What's the probability that I have to wait for at least 5 more?

8.7 The expected lifetime of a radioactive isotope, called probabilitium, is 1 year. Assuming that its lifetime follows an exponential distribution, what is its half-life?

8.8 Selena is watching shooting stars in the sky. On a typical summer night, many meteors reach the earth, and Selena sees each of them with a small probability, independently of the others. Typically, she sees around 3 in half an hour. She starts watching the sky on August 19th, 22:00.

(a) What is the probability that she doesn't see a single shooting star between 22:00 and 22:25?

(b) Let T be the time in minutes that Selena needs to wait to see the first shooting star. Calculate the CDF and PDF of T?

(c) What is the lesson?

8.9 The lifetime of the "Mehr Licht" light bulb is exponentially distributed. According to the manufacturer's measurements, 90 percent of light bulbs last for at least one year. How long should the official warranty period of a light bulb be if the company wants no more than 1 percent of customers to complain?

HW 8.10 (3 points) A random glass in the university cafeteria breaks at an exponentially distributed time, with mean 24 months. What is the probability that:

(a) Out of 6 glasses, at most 3 breaks within a year?

(b) Out of 500 glasses, at most 210 breaks within a year? (Give not just a formula, but a numerical value!)

♣ **8.11** (3 points) In Monte Carlo integration, we are given a continuous function $f : [0,1] \to [0,1]$, and we would like to estimate the value of $\int_0^1 f(x)\mathrm{d}x$. If we have independent $Uni[0,1]$ variables U and V, then (U,V) is a uniform random point in the unit square $[0,1]^2$. Generate n independent samples (U_i, V_i), $i = 1, \ldots, n$, and let X_n denote the number of pairs that fall below the graph of f, i.e., for which $V_i < f(U_i)$. Then, the integral is estimated by X_n/n. How large n should be, if we want to get an error larger than 0.1 only with a probability at most 0.02, if the function is

(a) $f(x) = x^3$;

(b) f is not known?

HW 8.12 (3 points) In Budapest, 43% of the citizens would ban smoking in public places. Approximate the probability that out of n people at least 40% of them supports this ban if

(a) $n = 11$,

(b) $n = 101$,

(c) $n = 1001$.

(d) How many people should we ask to guarantee that at least 40% of them will support the ban with probability 95%?

$\Phi(z)$

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000