

NAME: NEPTUN CODE:

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Probability Theory 1st midterm, 21th October 2021.

Working time: 45 minutes. Only simple, non-programmable calculators are allowed.

Maximum score: 24 pont, but we consider 20 points already as 100%.

1. On the occasion of the 240th anniversary of the founding of the university, a prize competition is organized. One can find a letter from the english alphabet in the cap of every bottle of PROBE drink bought in the buffet of building Ξ (each letter with probability $\frac{1}{26}$, independently of other bottles). Everyone who collects at least one bottle cap with B, one with M and one with E will win a promotional T-shirt. I buy 30 bottles of PROBE. What is the probability that I manage to collect all letters necessary to win the prize? (10 points)
2. Alice and Bella have a red and a blue dice. The red die is fair but the blue is biased, the probability of rolling a 6 with it is p ($0 < p < 1$). They throw both of the dice, if the red is 6 and the blue is not, then Alice wins. If the blue is 6 and the red is not, then Bella wins. If both or none is 6, then they roll the two dice again. They keep playing this game until one of them wins. Write p_A and p_B for the probability of Alice or Bella winning, respectively. We know that $p_B = 2p_A$.
 - (a) $p = ?$ (4 points) $p_A = ?$ (2 points)
 - (b) Let X denote the total number of throws (here one throw means throwing with both of the dice at once). $\mathbb{E}X = ?$ (4 points)

Bonus: Consider a usual 5 out of 90 lottery draw (90 different balls numbered from 1 to 90, 5 is drawn at once). Let X denote the smallest number drawn. Show that $\mathbb{E}X = \frac{91}{6}$. (4 points)