NAME: .... NEPTUN CODE: ....

Probability Theory 1st midterm, 20th October 2022.
Working time: 45 minutes. Only simple, non-programmable calculators are allowed.
Maximum score: 24 points, but we consider 20 points already as 100%.

1. The random numbers X_1, X_2, X_3, X_4, X_5 may take any value from $1, 2, ..., 9$ with probability $\frac{1}{9}$, independently of each other. Let Y be the product of these numbers (that is $Y = X_1 \cdot X_2 \cdot X_3 \cdot X_4 \cdot X_5$).

    (a) What is the probability that Y is odd? (2 points)

    (b) What is the probability that Y is divisible by 70? (8 points) (Note: $70 = 2 \cdot 5 \cdot 7$.)

2. A greengrocer has been buying walnuts from the same farm for a long time. According to experience, on average there is only 1 maggoty piece in 3 kg of walnuts, while the probability that there is no rancid piece in 4 kg of walnuts is 0.135335. We assume that being rancid or maggoty are independent phenomena.

    (a) My neighbour bought 8 kg of walnuts. What is the probability that he will find at least 2 rancid pieces? (4 points)

    (b) I would like to buy a bag of wallnuts without any rancid or maggoty pieces in it. At most how much walnut can I buy if I want to succeed with at least 70% chance? (6 points)

Bonus: In a casino, customers can try the following gambling machine. It generates a random permutation $(a_1, a_2, ..., a_N)$ of the numbers $1, 2, ..., N$ (each permutation has the same probability). There is a record at the k-th position if $a_k > a_i$ for every $i < k$. We earn 10 Ft after each record we got, and we have to pay 1000 Ft to play this game. At most how much can N be if this game favours the casino? (4 points)