

NAME: ..... NEPTUN-CODE: ..... MAJOR: .....

**Probability Theory 1 2nd midterm, 8th December 2023.**

**14:15–15:00**

*Working time: 45 minutes. Only a simple scientific, non-programmable calculator can be used.  
Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.*

1. The shape of a simplified darts board is a circle of radius 5 (suppose that the center of the board is the origin). It is divided as follows: in the middle, there is the bull's eye which is a small circle of radius 1 centered at the same point as the board, and it worths 25 points. The remaining part (which is an annulus) is divided into 20 equal parts such that the  $x$ -axis is a borderline, and the parts worth 1, 2, ..., 20 points respectively arranged counter clockwise beginning from the one which touches the  $x$ -axis from above. A player throws a dart with distribution of the pair of the random variables  $(X, Y)$ , where  $X$  and  $Y$  have normal distribution  $N(0, 5)$ , and  $X$  and  $Y$  are independent.
  - (a) What is the probability that the player hits the bull's eye? (3 points)
  - (b) What is the probability that the player hits the piece worths 20 points? (4 points)
2. I usually set up my alarm clock at 7:05. Then my morning preparation takes  $X$  minutes, where  $X$  is uniformly distributed on the interval  $[15, 30]$ . After that my trip from my flat to the university is  $Y$  minutes long, where  $Y$  is also uniformly distributed on the interval  $[15, 30]$ . The time of my preparation and my trip are independent. What is the probability that I arrive before 8:00? (6 points)

Extra: What is the density function of the time of my arrival? (4 points)
3. The manufacturing line of the company of Samuel Sung produces 100 televisions per day. Each product is scrap with probability of 0.1 independently. A day is called a sad day if the number of scrap products exceeds 15. The line operates 365 days a year.
  - (a) What is the probability that a day is a sad day? (3 points)
  - (b) What is the probability that there are at least 21 sad days in a year? (4 points)

