

PROB. THEO. 1, MAKE-UP MIDTERM 1., 2024.10.22.

$$\textcircled{1} \text{ a) } 1 - \left(\frac{2}{3}\right)^6$$

$$\text{b) } \frac{\binom{6}{2} \cdot \binom{4}{2}}{3^6} = \frac{6!}{2! \cdot 2! \cdot 2! \cdot 3^6}$$

c)  $A_i := \{ \text{I DID NOT COLLECT LETTER } i \}$

$$P(\text{I DID NOT WIN}) = P(A_W \cup A_I \cup A_N) =$$

$$P(A_W) + P(A_I) + P(A_N) - P(A_W \cap A_I) -$$

$$P(A_W \cap A_N) - P(A_I \cap A_N) + P(A_W \cap A_I \cap A_N)$$

$$= \left(\frac{2}{3}\right)^6 + \left(\frac{2}{3}\right)^6 + \left(\frac{2}{3}\right)^6 - \left(\frac{1}{3}\right)^6 - \left(\frac{1}{3}\right)^6 - \left(\frac{1}{3}\right)^6 + 0$$

$$= 3 \cdot \left(\frac{2}{3}\right)^6 - 3 \cdot \left(\frac{1}{3}\right)^6$$

BONUS:  $X_i := \mathbb{1}[\text{I COLLECTED LETTER } i]$

$E(\text{NUMBER OF COLLECTED DIFFERENT LETTERS}) =$

$$E(X_W + X_I + X_N) = E(X_W) + E(X_I) + E(X_N) =$$

$$= 3 \cdot \left(1 - \left(\frac{2}{3}\right)^6\right)$$

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$$\textcircled{2} \text{ a) } \frac{13 \cdot 4 \cdot 3}{52 \cdot 51} = \frac{3}{51}$$

b) A = FIRST CARD IS LARGER

B = SECOND " - " - " -

C = THEY WORTH THE SAME

$$P(A) = P(B) \text{ BY SYMMETRY. } P(C) = \frac{3}{51}$$

$$P(A) + P(B) + P(C) = 1.$$

$$\text{THUS } P(A) = \frac{1}{2} \cdot \left(1 - \frac{3}{51}\right) = \frac{24}{51} = \frac{8}{17}$$

c) C := {THE SUM OF VALUES IS 6}

D := {FIRST CARD IS AN ACE}

$$P(D|C) = \frac{P(D \cap C)}{P(C)}$$

$$= \frac{(4 \cdot 4) / (52 \cdot 51)}{(4 \cdot 4 \cdot 4 + 4 \cdot 3) / (52 \cdot 51)} = \frac{16}{76} = \frac{4}{19}$$