PROB. THEO. 1., 2ND MIDTERM, 2024.11.19.

$\boxed{\text{GROUP A}}$

(1) RS: $X \sim EXP\left(\frac{1}{70}\right)$

CU: $Y \sim EXP\left(\frac{1}{65}\right)$

a) $P(\text{LOSER THROWS} \geq 60) = P(\text{BOTH} \geq 60) =$

$P(X \geq 60, Y \geq 60) \overset{\text{INDEP.}}{=}$

$P(X \geq 60) \cdot P(Y \geq 60) = e^{-\frac{60}{70}} \cdot e^{-\frac{65}{70}} = 0.1677$

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b) $f(x,y) = f_X(x) \cdot f_Y(y) = \frac{1}{70} \cdot e^{-x/70} \cdot \frac{1}{65} \cdot e^{-y/65}$

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c) $P(\text{RS WINS}) = P(X \geq Y) = \iint_{x>y} f(x,y)\,dx\,dy$

$= \int_0^\infty \int_y^\infty \frac{1}{70} \cdot e^{-x/70} \cdot \frac{1}{65} \cdot e^{-y/65}\,dx\,dy =$

$= \int_0^\infty \frac{1}{65} \cdot e^{-y/65} \cdot e^{-y/70}\,dy = \frac{1/65}{1/65 + 1/70} = 0.5185$

BONUS: $E(\sqrt{X}) = \int_0^\infty \lambda \cdot e^{-\lambda x} \cdot \sqrt{x}\,dx =$

$\boxed{y = \sqrt{x} \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad 2\sqrt{x}\,dy = dx}$

$\int_0^\infty \lambda \cdot e^{-\lambda \cdot y^2} \cdot 2y^2\,dy =$ ☺ LET $\boxed{\lambda = \frac{1}{2} \cdot \frac{1}{\sigma^2}}$

THUS $\boxed{\sigma = \frac{1}{\sqrt{2\lambda}}}$

$$☺ = 2\lambda \cdot \int_0^{\infty} e^{-y^2/2\sigma^2} \cdot y^2\, dy = \lambda \cdot \int_{-\infty}^{\infty} e^{-y^2/2\sigma^2} y^2\, dy$$

$$= \lambda \cdot \sqrt{2\pi} \cdot \sigma \cdot \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \cdot e^{-y^2/2\sigma^2} \cdot y^2\, dy}_{= \sigma^2} =$$

VARIANCE OF $\mathcal{N}(0, \sigma^2)$

$$= \lambda \cdot \sqrt{2\pi} \cdot \sigma^3 = \lambda \cdot \sqrt{2\pi} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot \lambda^{-\frac{3}{2}} = \frac{\sqrt{\pi}}{2 \cdot \sqrt{\lambda}}$$

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2) $A \sim \mathcal{N}(100, 4^2)$, $B \sim \mathcal{N}(90, 3^2)$

a) $\boxed{C := A - B}$ $C \sim \mathcal{N}(\underbrace{10}_{\mu}, \underbrace{4^2 + 3^2}_{5^2 = \sigma^2})$

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b) $\mathbb{P}(A > B) = \mathbb{P}(C > 0) = \mathbb{P}\left(\frac{C - 10}{5} > \frac{0 - 10}{5}\right)$

$= 1 - \Phi(-2) = \Phi(2) = 0.9772$

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c) $\mathbb{P}($B IS PROFITABLE TODAY$) = \mathbb{P}(B \geq 92) =$

$= \mathbb{P}\left(\frac{B - 90}{3} \geq \frac{92 - 90}{3}\right) = 1 - \Phi\left(\frac{2}{3}\right) = 1 - 0.7486 =$

$= 0.2514 =: p$

$X :=$ NUMBER OF PROFITABLE DAYS THIS YEAR

$X \sim BIN(365, p)$

DE MOIVRE – LAPLACE

$$\mathbb{P}(X \geq 100) = \mathbb{P}\left(\frac{X - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}} \geq \frac{100 - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}}\right) \approx$$

$$1 - \Phi\left(\frac{100 - 365 \cdot p}{\sqrt{365 \cdot p \cdot (1-p)}}\right) = 1 - \Phi(0.99) = 0.1611$$

$$100 - 365 \cdot p = 8.239$$

$$365 \cdot p \cdot (1-p) = 68.69$$

$$\sqrt{365 \cdot p \cdot (1-p)} = 8.288$$