(A) 2. ZH Valószínűségszámítás, 2024 nov. 25.

(1) A, B, C, D, E

$X_n = \{ \text{hány nap mosogat A n nap alatt} \} \sim Bin(n, \frac{1}{5})$.

$\mathbb{P}(X_n \leq 0{,}22n) \geq 0{,}99$ $\quad E(X_n) = \frac{n}{5}$, $\mathbb{V}(X_n) = n\frac{4}{25}$

de-Moivre-Laplace $Z := \frac{X_n - 0{,}2n}{\sqrt{n\frac{4}{25}}} \sim N(0,1)$

$0{,}99 \leq \mathbb{P}(X_n \leq 0{,}22n) = \mathbb{P}\left(Z \leq \frac{0{,}02\,n}{\sqrt{n}\,\frac{2}{5}}\right) =$

$= \Phi(\sqrt{n}\;0{,}05)$

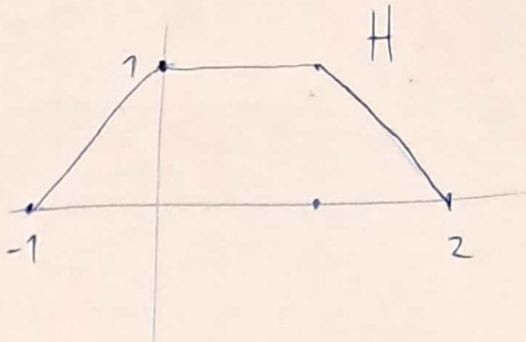
$2{,}33 \leq \sqrt{n} \cdot 0{,}05$

$46{,}6 \leq \sqrt{n}$

$2171{,}56 \leq n$. Legalább 2172 nap kell.

CamScanner

② $(X, Y) \sim Uni(H)$



$f(x,y) = \frac{\mathbb{1}_H}{2}$

a) $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \frac{1}{2}\int_{-\infty}^{\infty} \mathbb{1}_H\,dx = \frac{1}{2}\int_{y-1}^{2-y} dx =$

$= \frac{3-2y}{2}\,\mathbb{1}_{y\in[0,1]}$

b) $\mathbb{P}(X>0 \mid Y=\frac{1}{2}) = \int_0^{\infty} f_{X|Y}(x,\frac{1}{2})\,dx = \int_0^{\infty} \frac{f(x,\frac{1}{2})}{f_Y(\frac{1}{2})}\,dx =$

$= \frac{\frac{1}{2}\int_0^{\frac{3}{2}} dx}{1} = \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$

③ 5 féle zokni, 7 db zokni

$\xi = \{$ hány féle zoknit gyűjtöttem össze $\}$

$X_i = \mathbb{1}_{A_i}$ $\qquad A_i = \{$ összegyűjtöttem az i. féle zoknit $\}$

$\xi = \sum_{i=1}^{5} X_i$

$\mathbb{E}(X_i) = \mathbb{P}(A_i) = 1-\mathbb{P}(A_i^c) = 1-\left(\frac{4}{5}\right)^7$

$\Rightarrow \mathbb{E}(\xi) = \sum_{i=1}^{5} \mathbb{E}(X_i) = 5\left(1-\left(\frac{4}{5}\right)^7\right)$

B'onyz: $\mathbb{V}(\xi) = Cov(\xi, \xi) = Cov\left(\sum_{i=1}^{5} X_i, \sum_{j=1}^{5} X_j\right) =$

$$= \sum_{i,j=1}^{5} Cov(X_i, X_j)$$

I, $i=j$ $\quad Cov(X_i, X_j) = \mathbb{V}(X_i) = \mathbb{E}(X_i^2) - \mathbb{E}(X_i)^2 =$

$$= \mathbb{P}(A_i) - \mathbb{P}(A_i)^2 = \mathbb{P}(A_i)\mathbb{P}(A_i^c) = \left(1-\left(\frac{4}{5}\right)^7\right)\left(\frac{4}{5}\right)^7$$

5 ilyen egyet van

II, $i \neq j$ $\quad Cov(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j) =$

$$= \mathbb{P}(A_i \cap A_j) - \mathbb{P}(A_i)^2$$

$$\mathbb{P}(A_i \cap A_j) = 1 - \mathbb{P}(A_i^c \cup A_j^c) \overset{szita}{=} 1 - \mathbb{P}(A_i^c) - \mathbb{P}(A_j^c) + \mathbb{P}(A_i^c \cap A_j^c)$$

$$= 1 - \left(\frac{4}{5}\right)^7 - \left(\frac{4}{5}\right)^7 + \left(\frac{3}{5}\right)^7$$

$\binom{5}{2}$ ilyen egyet van

$$\mathbb{V}(\xi) = 5 \cdot \left(1-\left(\frac{4}{5}\right)^7\right)\left(\frac{4}{5}\right)^7 + \binom{5}{2}\left(1 - 2\left(\frac{4}{5}\right)^7 + \left(\frac{3}{5}\right)^7 - \left(1-\left(\frac{4}{5}\right)^7\right)^2\right) =$$

$$= 5\left(1-\left(\frac{4}{5}\right)^7\right)\left(\frac{4}{5}\right)^7 + \binom{5}{2}\left(\left(\frac{3}{5}\right)^7 - \left(\frac{4}{5}\right)^{14}\right)$$