NAME: ..... NEPTUN-CODE: ..... TUTOR: .....
Group A

Probability Theory 1 1st midterm, 8th October 2024. 16:15–17:00

Working time: 45 minutes. Only simple scientific, non-programmable calculators are allowed.
Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.

1. There are three universities in Random City: the Eltie, the Avegare and the Bhad. Only 20% of the students studies at Eltie, however, it is the hardest to get a good grade there, only 30% of the graduated students get a good grade, 70% gets a bad grade. The 50% of the students studies at the Avegare, and 50% of the graduated students get a good grade there, the rest get a bad grade. The remaining students are studying at the Bhad University, there the probability to graduate with a good grade is 70%. The grades of students at each university are independent from each other.

   A company announced a new job advertisement, and they got applications from three candidates.

   (a) What is the probability that the first applicant graduated with a good grade? (2 points)

   (b) It turns out that all three applicants are from the same university. What is the probability that all of them graduated with a good grade? (4 points)

   (c) The company chose the first applicant who graduated with a good grade. What is the probability that she graduated at the Eltie? (4 points)

2. In a high school, the students have two different official dresses (white and blue) which they have to wear during the schoolday. Every day every student chooses which dress he/she wears randomly, independently from his/her earlier choices and from the choices of other students.

   (a) What is the probability that in a class of 20 students, the number of blue and white dresses is the same today? (2 points)

   (b) What is the probability that at least one student wears a blue dress today? (3 points)

   (c) What is the probability that on a week (of 5 days) there will be exactly two days when exactly two students wear blue dress in the class? (5 points)

Bonus: To collect enough money for an excursion of the class, the students decided that they will do the following: each member of the group of the students whose dress has the majority pays \$1 to the class' fund. (In case of tie, i.e. equal number of blue and white dresses, there is no transaction.) What is the expected value of the collected money on a day? (We expect a specific numerical value not a sum with a proper verification.) (4 points)

NAME: ..... NEPTUN-CODE: ..... TUTOR: .....
Group B

Probability Theory 1 1st midterm, 8th October 2024. 16:15–17:00

Working time: 45 minutes. Only simple scientific, non-programmable calculators are allowed.
Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.

1. There are three light bulb factories in Random City: the Execlent, the Avegare and the Bhad. Only 15% of the bulbs are produced by Execlent, however, those are in a very good quality, only 10% of the bulbs are faulty, 90% has good quality. The 50% of the bulbs are produced in the Avegare, and 20% of the bulbs are faulty, the rest are of good quality. The remaining bulbs are produced in the Bhad Factory, there the probability to produce a faulty bulb is 40%. The quality of the bulbs at each factory are independent from each other.

   I bought four bulbs in a local shop.

   (a) What is the probability that the first bulb is not faulty? (2 points)

   (b) It turns out that all four bulbs are from the same factory. What is the probability that all of them are faulty? (4 points)

   (c) I tried out the first bulb and it was working properly. What is the probability that it was produced in the factory Execlent? (4 points)

2. Peter always keeps 30 coins in his pocket. Peter is an extremely clumsy guy. Every day, when he wants to pay in a shop and he takes out his wallet from his pocket, all the coins fall out. The coins fall independently and are unbiased.

   (a) What is the probability that the number of heads and tails on the ground are equal? (2 points)

   (b) What is the probability that there are at least two heads on the ground? (3 points)

   (c) What is the probability that on a week (of 5 days) there will be exactly three days when exactly 10 heads will be on the ground? (5 points)

Bonus: The friends of Peter fed up with that and they decided to take away the money from Peter in the following way: They take away the coins from Peter which had the majority on the ground. That is, if there are strictly more heads on the ground than tails they take away the coins with head outcome, if there are strictly more tails than heads they take away the coins with tail outcome. In case of tie, i.e. equal number of heads and tails, they don't take away the money. What is the expected value of the collected money? (We expect a specific numerical value not a sum with a proper verification.) (4 points)