2024.10.08. MIDTERM 1, GROUP A

① a) $\mathbb{P}(\text{GOOD GRADE}) =$ ← LAW OF TOTAL PROBAB.

$= \underbrace{\mathbb{P}(\text{GOOD} \mid E)}_{0.3} \cdot \underbrace{\mathbb{P}(E)}_{0.2} + \underbrace{\mathbb{P}(\text{GOOD} \mid A)}_{0.5} \cdot \underbrace{\mathbb{P}(A)}_{0.5} + \underbrace{\mathbb{P}(\text{GOOD} \mid B)}_{0.7} \cdot \underbrace{\mathbb{P}(B)}_{0.3}$

$= 0.52$

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b) $\mathbb{P}(\text{GOOD}, \text{GOOD}, \text{GOOD} \mid \text{SAME}) = \dfrac{\mathbb{P}(\text{GGG AND SAME})}{\mathbb{P}(\text{SAME})} =$

$= \dfrac{(0.3)^3 \cdot (0.2)^3 + (0.5)^3 \cdot (0.5)^3 + (0.7)^3 \cdot (0.3)^3}{(0.2)^3 + (0.5)^3 + (0.3)^3} = 0.157$

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c) $\mathbb{P}(E \mid G) = \dfrac{\mathbb{P}(E \cap G)}{\mathbb{P}(G)} = \dfrac{0.3 \cdot 0.2}{0.52} =$

$= 0.1154$

a)

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② X = NUMBER OF BLUE DRESSES ON A GIVEN DAY

$X \sim \text{BIN}\left(20, \frac{1}{2}\right)$

T.B.C.

a) $\mathbb{P}(X=10) = \binom{20}{10} \cdot \left(\frac{1}{2}\right)^{10} \cdot \left(1-\frac{1}{2}\right)^{10} \approx 0.176$

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b) $\mathbb{P}(X \geq 1) = 1 - \left(\frac{1}{2}\right)^{20}$

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c) $\mathbb{P}(X=2) = \binom{20}{2} \cdot \left(\frac{1}{2}\right)^{20} = q$

$\mathbb{P}($ 2 DAYS WHEN 2 WEAR BLUE $) =$

$= \binom{5}{2} \cdot q^2 \cdot (1-q)^3$

BONUS: $\mathbb{E}(\text{MONEY}) = m_1 + m_2$ WHERE

$m_1 = \sum_{k=11}^{20} k \cdot \mathbb{P}(X=k)$, $m_2 = \sum_{k=0}^{9} (20-k) \cdot \mathbb{P}(X=k)$

BY SYMMETRY, WE HAVE $m_1 = m_2$

$m_1 = \sum_{k=11}^{20} k \cdot \binom{20}{k} \cdot \left(\frac{1}{2}\right)^{20} = \sum_{k=11}^{20} k \cdot \frac{(20)!}{k!\,(20-k)!} \cdot 2^{-20} =$

$= \sum_{k=11}^{20} \frac{20!}{(k-1)!\,(20-k)!} \cdot 2^{-20} = 20 \cdot \sum_{k=11}^{20} \binom{19}{k-1} \cdot 2^{-20} =$

$20 \cdot \sum_{\ell=10}^{19} \binom{19}{\ell} \cdot 2^{-20} = 10 \cdot \mathbb{P}(Y \geq 10) = 10 \cdot \frac{1}{2} = 5$

WHERE $Y \sim BIN\left(19, \frac{1}{2}\right)$, THUS $\mathbb{E}(\text{MONEY}) = 10$