NAME: .... NEPTUN-CODE: .... MAJOR: ....

Probability Theory 1, 1st midterm, 19th October 2023. Group A, 8:05–8:50

Working time: 45 minutes. Only a simple scientific, non-programmable calculator can be used. Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.

1. There are 10 classrooms in building η of BME, numbered 001, 002, $\dots 010$. On Monday evening, just before 9.00pm, each classroom has 3-3 lights turned on. These lights can be turned off by 30 switches located on a central switch box in the janitor's closet. Each light is assigned to exactly one switch, but there are no markings on the switch box, so it is not known which light is controlled by which switch. At exactly 9.00pm, the janitor randomly selects and turns off exactly 10 out of the 30 switches, to save energy. At that time, I am in classroom 001 at a probability consultation. For the following questions, we do not expect a numerical value, it is enough to express the answer in terms of binomial coefficients.
   (a) (2 points) What is the probability that all three lights in room 001 remain switched on?
   (b) (2 points) What is the probability that room 001 will be completely dark (i.e., the night watchman turns off all three lights in the room)?
   (c) (6 points) What is the probability that none of the 10 classrooms will turn completely dark (in the sense of question (b))?

2. A large research institute uses four servers for serious calculations, called Euler, Fermat, Gauss and Hilbert. Researchers can send computational problems to these servers every weekday between 6.00am and 10.00pm. There are many researchers working at the institute, and on a given day each of them (independently of each other and of the other days) may send a computation to the servers with a small probability. The servers process the incoming problems according to the following protocol: each day, the first incoming problem is given to Euler, the second to Fermat, the third to Gauss, the fourth to Hilbert, the fifth to Euler, the sixth to Fermat, and so on. This protocol has been in use for a long time, and we know that in about one twentieth of the weekdays, there is no computational problem sent to the servers.
   (a) (5 points) What is the probability that on October 19, 2023, each of the four servers will receive exactly one new computational problem?
   (b) (5 points) What is the probability that on October 20, 2023, Fermat will receive at least two new computational problems?

Bonus (4 points) What is the probability that on October 24, 2023, Euler, Fermat, Gauss and Hilbert will receive the same number of new computational problems? (*Note*: we expect not only an infinite series, but also a specific numerical value as an answer to this question.)