2023.10.19 SOLUTIONS GROUP A

1 a) $$\frac{\binom{27}{10}}{\binom{30}{10}} = \frac{27}{30} \cdot \frac{26}{29} \cdot \ldots \cdot \frac{18}{21}$$

b) $$\frac{\binom{27}{7}}{\binom{30}{10}}$$

c) LET $A_i := \{\text{ROOM } i \text{ TURNS DARK}\}$

$$\underbrace{\mathbb{P}\left(\bigcap_{i=1}^{10} A_i^c\right)}_{= 1 - \mathbb{P}\left(\bigcup_{i=1}^{10} A_i\right)} = ?$$

$$\mathbb{P}\left(\bigcup_{i=1}^{10} A_i\right) = \sum_{\substack{I \subseteq [10] \\ I \neq \emptyset}} (-1)^{|I|+1} \cdot \mathbb{P}\left(\bigcap_{i \in I} A_i\right) = \star$$

$$\mathbb{P}\left(\bigcap_{i \in I} A_i\right) = \frac{\binom{30-3\cdot|I|}{10-3\cdot|I|}}{\binom{30}{10}}$$ IF $|I| \leq 3$, OTHERWISE ZERO

$$\star = \sum_{\substack{I \subseteq [10] \\ 0 < |I| \leq 3}} (-1)^{|I|+1} \cdot \frac{\binom{30-3\cdot|I|}{10-3\cdot|I|}}{\binom{30}{10}} =$$

$$= \binom{10}{1} \cdot \frac{\binom{27}{7}}{\binom{30}{10}} - \binom{10}{2} \cdot \frac{\binom{24}{4}}{\binom{30}{10}} + \binom{10}{3} \cdot \frac{\binom{21}{1}}{\binom{30}{10}}$$

2) X = NUMBER OF PROBLEMS SENT TO SERVERS ON A FIXED DAY

$$\boxed{X \sim POI(\lambda)} \qquad \frac{1}{20} = \mathbb{P}(X=0) = e^{-\lambda}$$

THUS $\boxed{\lambda = \ln(20)}$

a) $\mathbb{P}(X=4) = e^{-\lambda} \cdot \frac{\lambda^4}{4!} = \frac{1}{20} \cdot \frac{\ln(20)^4}{24}$

b) $\mathbb{P}(X \geq 6) = 1 - \mathbb{P}(X \leq 5) =$

$$1 - \sum_{r=0}^{5} \frac{1}{20} \cdot \frac{\ln(20)^r}{r!}$$

BONUS: $\mathbb{P}(X \text{ IS DIVISIBLE BY } 4) = ?$

$P_r = \mathbb{P}(X = r)$, $r = 0, 1, 2, \ldots$

$$\boxed{\sum_{r=0}^{\infty} P_r = 1} \quad \boxed{\sum_{r=0}^{\infty} (-1)^r \cdot P_r = \sum_{r=0}^{\infty} e^{-\lambda} \cdot \frac{(-\lambda)^r}{r!} = e^{-2\lambda}}$$

$$\boxed{\sum_{r=0}^{\infty} (i)^r \cdot P_r = e^{-\lambda} \cdot e^{i \cdot \lambda} = e^{-\lambda} \cdot (\cos(\lambda) + i \cdot \sin(\lambda))}$$

$$\boxed{\sum_{r=0}^{\infty} (-i)^r \cdot P_r = e^{-\lambda} \cdot (\cos(\lambda) - i \cdot \sin(\lambda))}$$

T.B.C.

$$? = \sum_{x=0}^{\infty} \mathbb{1}[4|x] \cdot p_x =$$

$$= \sum_{x=0}^{\infty} \frac{1}{4} \cdot \left(1 + (-1)^x + (i)^x + (-i)^x\right) \cdot p_x$$

$$= \frac{1}{4} \cdot \left(1 + e^{-2\lambda} + 2 \cdot e^{-\lambda} \cdot \cos(\lambda)\right)$$