VS21, ZH2, 2024.11.25., B CSOPORT

① $A \sim N(2 \cdot 10^6, (10^5)^2)$

$B \sim N(3 \cdot 10^6, (2 \cdot 10^5)^2)$

$K \sim N(4{,}5 \cdot 10^6, (2 \cdot 10^5)^2)$

PROFIT: $P = A + B - K$

$E(P) = 2 \cdot 10^6 + 3 \cdot 10^6 - 4{,}5 \cdot 10^6 = 0{,}5 \cdot 10^6 = 5 \cdot 10^5$

$Var(P) = (10^5)^2 + (2 \cdot 10^5)^2 + (2 \cdot 10^5)^2 = 9 \cdot (10^5)^2$

$P \sim N(5 \cdot 10^5, 9 \cdot (10^5)^2)$

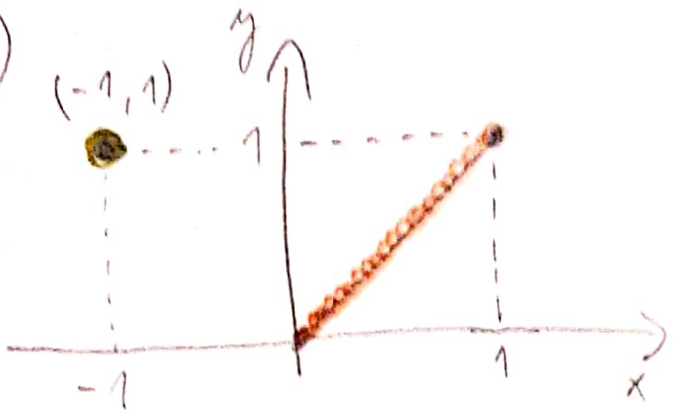
$$P(P \geq 2 \cdot 10^5) = P\left(\frac{P - 5 \cdot 10^5}{\sqrt{9 \cdot (10^5)^2}} \geq \frac{2 \cdot 10^5 - 5 \cdot 10^5}{\sqrt{9 \cdot (10^5)^2}}\right) =$$

$$= 1 - \Phi\left(\frac{2 \cdot 10^5 - 5 \cdot 10^5}{\sqrt{9 \cdot (10^5)^2}}\right) = 1 - \Phi(-1) = \Phi(1)$$

$$= 0.8413$$

1.OLDAL

(2)



A NARANCS SZÍNŰ SZAKASZON EGYENLETES ELOSZLÁSÚ PONT TÁVOLSÁGA A ZÖLD PONTTÓL D.

$$F(x) = \mathbb{P}(D \le x) = \begin{cases} 0, \text{ HA } x \le \sqrt{2} \\ \text{(A)}, \text{ HA } x \in (\sqrt{2}, 2) \\ 1, \text{ HA } x \ge 2 \end{cases}$$

$$D = \sqrt{(u-(-1))^2 + (u-1)^2} = \sqrt{2\cdot u^2 + 2}$$

$$\text{(A)} = \mathbb{P}\left(\sqrt{2\cdot u^2 - 2} \le x\right) = \mathbb{P}\left(2\cdot u^2 - 2 \le x^2\right) =$$

$$= \mathbb{P}\left(u^2 \le \frac{x^2}{2} - 1\right) = \mathbb{P}\left(u \le \sqrt{\frac{x^2}{2} - 1}\right) = \sqrt{\frac{x^2}{2} - 1}$$

HISZEN HA $x \in (\sqrt{2}, 2)$, AKKOR $\sqrt{\frac{x^2}{2} - 1} \in (0,1)$

$$\mathbb{P}(D \ge \tfrac{3}{2}) = 1 - F(\tfrac{3}{2}) = 1 - \sqrt{\frac{(3/2)^2}{2} - 1} =$$

$$= 1 - \frac{1}{\sqrt{8}} = 0.64644$$

$$f(x) = F'(x) = \begin{cases} 0, \text{ HA } x \le \sqrt{2} \\ \text{☺}, \text{ HA } x \in (\sqrt{2}, 2) \\ 0, \text{ HA } x \ge 2 \end{cases}$$

$$\text{☺} = \frac{d}{dx}\sqrt{\frac{x^2}{2} - 1} = \frac{1}{2\sqrt{\frac{x^2}{2} - 1}} \cdot x$$

2. OLDAL

(3) $\xi_i := \mathbb{1}[A_i]$

$A_i := \{$ AZ i-EDIK FÁRA MÁSZOTT MAJOM $\}$

$$\boxed{\xi = \sum_{i=1}^{54} \xi_i} \qquad \mathbb{E}(\xi) \overset{\text{LIN.}}{=} \sum_{i=1}^{54} \mathbb{E}(\xi_i) =$$

$$= \sum_{i=1}^{54} \mathbb{P}(A_i) = 54 \cdot \left(1 - \left(\frac{53}{54}\right)^{43}\right) \approx 29.827$$

BÓNUSZ: ξ SZÓRÁSA $= \sqrt{\mathrm{Var}(\xi)} = \sigma$

$\mathrm{Var}(\xi) = \mathbb{E}(\xi^2) - \mathbb{E}(\xi)^2 =$ ☺

$$\mathbb{E}(\xi^2) = \mathbb{E}\left(\sum_{i,j=1}^{54} \xi_i \cdot \xi_j\right) = \sum_{i,j=1}^{54} \mathbb{P}(A_i \cap A_j)$$

$$= \underbrace{54 \cdot \mathbb{P}(A_1)}_{\mathbb{E}(\xi)} + 54 \cdot 53 \cdot \mathbb{P}(A_1 \cap A_2)$$

$$= 1 - \mathbb{P}(A_1^c \cup A_2^c) =$$

$$= 1 - \left(\mathbb{P}(A_1^c) + \mathbb{P}(A_2^c) - \mathbb{P}(A_1^c \cap A_2^c)\right) =$$

$$= 1 - 2 \cdot \left(\frac{53}{54}\right)^{43} + \left(\frac{52}{54}\right)^{43} = 0.302$$

$$\sigma = \sqrt{☺} = \sqrt{894.151 - (29.827)^2} \qquad 2.12$$

3. OLDAL