

$$1. \quad \left. \begin{array}{l} x^2 + xy = 210 \\ y^2 + xy = 231 \end{array} \right\} \Rightarrow \left. \begin{array}{l} (x+y)^2 = 441 \\ y \cdot (y+x) = 231 \end{array} \right\} \Rightarrow \begin{array}{l} \text{I. } x+y = 21, \quad 21y = 231 \\ \text{II. } x+y = -21, \quad -21y = 231 \end{array} \Rightarrow \begin{array}{l} y=11 \quad x=10 \\ y=-11 \quad x=-10 \end{array}$$

$$2. \quad \left. \begin{array}{l} xy + x + y = 29 \\ xy - 2x - 2y = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 3 \cdot (x+y) = 27 \\ 3xy = 60 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x+y=9 \\ xy=20 \end{array} \right\} \Rightarrow x \text{ és } y \text{ a } z^2 - 9z + 20 = 0 \text{ gyökei: } \begin{array}{l} x=4 \quad y=5 \\ x=5 \quad y=4 \end{array}$$

$$3. \quad \left. \begin{array}{l} (2x+y)^2 = 16 \\ x - \frac{1}{y} = \frac{1}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} |2x+y| = 4 \\ 2x = 1 + \frac{2}{y} \end{array} \right\} \Rightarrow \begin{array}{l} \text{I. } 2x+y = 4 \\ 4-y = 1 + \frac{2}{y} \Rightarrow y^2 - 3y + 2 = 0 \Rightarrow \begin{array}{l} y=1 \quad x=\frac{3}{2} \\ y=2 \quad x=1 \end{array} \\ \text{II. } 2x+y = -4 \\ -4-y = 1 + \frac{2}{y} \Rightarrow y^2 + 5y + 2 = 0 \Rightarrow \begin{array}{l} y = \frac{-5+\sqrt{17}}{2} \quad x = \frac{-3-\sqrt{17}}{4} \\ y = \frac{-5-\sqrt{17}}{2} \quad x = \frac{-3+\sqrt{17}}{4} \end{array} \end{array}$$

$$4. \quad \left. \begin{array}{l} x^2 - 6xy + 9y^2 = 25 \\ x + \frac{1}{y} = 9 \end{array} \right\} \Rightarrow \left. \begin{array}{l} |x-3y| = 5 \\ x = 9 - \frac{1}{y} \end{array} \right\} \Rightarrow \begin{array}{l} \text{I. } x-3y = 5 \\ 5+3y = 9 - \frac{1}{y} \Rightarrow 3y^2 - 4y + 1 = 0 \Rightarrow \begin{array}{l} y=1 \quad x=8 \\ y=\frac{1}{3} \quad x=6 \end{array} \\ \text{II. } x-3y = -5 \\ -5+3y = 9 - \frac{1}{y} \Rightarrow 3y^2 - 14y + 1 = 0 \Rightarrow y_{1,2} = \frac{14 \pm \sqrt{14^2 - 4 \cdot 3}}{6} = \frac{7 \pm \sqrt{49-3}}{3} \end{array} \Rightarrow \begin{array}{l} y = \frac{7+\sqrt{46}}{3} \quad x = 2 + \sqrt{46} \\ y = \frac{7-\sqrt{46}}{3} \quad x = 2 - \sqrt{46} \end{array}$$

$$5. \quad \left. \begin{array}{l} x + \frac{3}{4}y = 9 \\ \frac{x}{2} - \frac{2y}{3} = \frac{1}{3} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = 9 - \frac{3}{4}y \\ 9 - \frac{3}{4}y - \frac{4y}{3} = \frac{2}{3} \end{array} \right\} \Rightarrow \left. \begin{array}{l} 9 \cdot 12 - 9y - 16y = 8 \\ 25y = 100 \end{array} \right\} \Rightarrow \boxed{y=4 \quad x=6}.$$

$$6. \quad \left. \begin{array}{l} \frac{x+2}{3} - \frac{y-3}{4} = 3 \\ \frac{3}{x+2} - \frac{1}{y-3} = 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y-3 = \frac{x+2}{3} \\ \frac{x+2}{3} - \frac{x+2}{12} = 3 \\ \frac{x+2}{4} = 3 \end{array} \right\} \Rightarrow \boxed{x=10 \quad y=7}.$$

$$7. \quad \left. \begin{array}{l} \frac{1}{x-2y} - \frac{2}{2x-y} = 3 \\ -\frac{2}{x-2y} + \frac{5}{2x-y} = -5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} A := \frac{1}{x-2y} \\ B := \frac{1}{2x-y} \end{array} \right\} \Rightarrow \left. \begin{array}{l} A - 2B = 3 \\ -2A + 5B = -5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} B = 1 \\ A = 5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2x - y = 1 \\ x - 2y = \frac{1}{5} \end{array} \right\} \Rightarrow \boxed{x = \frac{3}{5} \quad y = \frac{1}{5}}$$

$$8. \left. \begin{cases} \frac{10}{x-5} + \frac{1}{y-2} = 1 \\ \frac{25}{x-5} + \frac{3}{y+2} = 2 \end{cases} \right\} \Rightarrow \left. \begin{cases} \frac{5}{y-2} - \frac{6}{y+2} = 1 \\ 5 \cdot (y+2) - 6 \cdot (y-2) = y^2 - 4 \end{cases} \right\} \Rightarrow \left. \begin{cases} y^2 + y - 26 = 0 \\ y = \frac{-1 \pm \sqrt{105}}{2} \end{cases} \right\} \Rightarrow \left. \begin{cases} \frac{10}{x-5} = 1 - \frac{2}{-5 \pm \sqrt{105}} \\ y-2 = \frac{-5 \pm \sqrt{105}}{2} \end{cases} \right\} \Rightarrow$$

$$\Rightarrow \text{I. } \left. \begin{cases} y = \frac{-1 + \sqrt{105}}{2} \\ \frac{10}{x-5} = \frac{-7 + \sqrt{105}}{-5 + \sqrt{105}} \end{cases} \right\} \left. \begin{cases} y = \frac{-1 + \sqrt{105}}{2} \\ x = 5 + 10 \cdot \frac{-5 + \sqrt{105}}{-7 + \sqrt{105}} = \frac{35}{2} + \frac{5\sqrt{105}}{14} \end{cases} \right\} \text{II. } \left. \begin{cases} y = \frac{-1 - \sqrt{105}}{2} \\ \frac{10}{x-5} = \frac{-7 - \sqrt{105}}{-5 - \sqrt{105}} \end{cases} \right\} \left. \begin{cases} y = \frac{-1 - \sqrt{105}}{2} \\ x = 5 + 10 \cdot \frac{-5 - \sqrt{105}}{-7 - \sqrt{105}} = \frac{35}{2} - \frac{5\sqrt{105}}{14} \end{cases} \right\}$$

$$9. \left. \begin{cases} y-x = 44 \\ \sqrt{\frac{6x}{x+y}} + \sqrt{\frac{x+y}{6x}} = \frac{5}{2} \end{cases} \right\} \Rightarrow \left. \begin{cases} A := \sqrt{\frac{6x}{x+y}} \\ A + \frac{1}{A} = \frac{5}{2} \end{cases} \right\} \Rightarrow \left. \begin{cases} 2A^2 - 5A + 2 = 0 \\ A = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4} \end{cases} \right\} \Rightarrow \left. \begin{cases} \text{I. } \sqrt{\frac{6x}{x+y}} = 2, & 6x = 4(x+y) \Rightarrow x = 2y \\ \text{II. } \sqrt{\frac{6x}{x+y}} = \frac{1}{2}, & 24x = x+y \Rightarrow y = 23x \end{cases} \right\}$$

$$\text{I. } y - 2y = 44, \quad y = -44 \quad \Rightarrow \quad \boxed{x = -88 \quad y = -44}$$

$$\text{II. } 23x - x = 44, \quad x = 2 \quad \Rightarrow \quad \boxed{x = 2 \quad y = 46}$$

$$10. \left. \begin{cases} 3^x + 4^y = 73 \\ 3^x \cdot 4^y = 576 \end{cases} \right\} \Rightarrow \left. \begin{cases} 3^x \text{ és } 4^y \text{ a } z^2 - 73z + 576 = 0 \text{ gyökei: } z = \frac{73 \pm \sqrt{73^2 - 4 \cdot 576}}{2} \\ 3^x = 64, \quad 4^y = 9 \quad \text{vagy} \quad 3^x = 9, \quad 4^y = 64 \end{cases} \right\} \Rightarrow \left. \begin{cases} \boxed{x = \log_3 64 \quad y = \log_4 9} \\ \boxed{x = 2 \quad y = 3} \end{cases} \right\}$$

$$11. \left. \begin{cases} 2x - 4\sqrt{x} + y - 4 = 0 \\ 5\sqrt{x} - y - 17 = 0 \end{cases} \right\} \Rightarrow \left. \begin{cases} \text{összeadva: } 2x + \sqrt{x} - 21 = 0 \\ \sqrt{x} \geq 0 \\ \sqrt{x} = \frac{-1 + \sqrt{1 + 4 \cdot 42}}{4} = 3 \end{cases} \right\} \Rightarrow \boxed{x = 9 \quad y = -2}$$

$$12. \left. \begin{cases} \frac{\sqrt{x}}{2} + y = 4 \\ y^2 - \sqrt{x} = 27 \end{cases} \right\} \Rightarrow \left. \begin{cases} 2y + \sqrt{x} - 8 = 0 \\ y^2 - \sqrt{x} - 27 = 0 \end{cases} \right\} \Rightarrow \left. \begin{cases} y^2 + 2y - 35 = 0 \\ y = 5, -7 \end{cases} \right\} \left. \begin{cases} \sqrt{x} = 8 - 2y \\ \text{csak } y = -7 \text{ lehetséges} \Rightarrow \\ \sqrt{x} = 8 + 14, \quad x = 22^2 \end{cases} \right\} \Rightarrow \boxed{y = -7 \quad x = 484}$$

$$13. \left. \begin{cases} 3^{\log_3 x} - 2^{\log_4 y} = 77 \\ 3^{\log_3 \sqrt{x}} - 2^{\log_{16} y} = 7 \end{cases} \right\} \Rightarrow \left. \begin{cases} x - \sqrt{y} = 77 \\ \sqrt{x} - 4\sqrt{y} = 7 \end{cases} \right\} \Rightarrow \left. \begin{cases} (\sqrt{x} - 4\sqrt{y}) \cdot (\sqrt{x} + 4\sqrt{y}) = 77 \\ \sqrt{x} + 4\sqrt{y} = 11 \end{cases} \right\} \Rightarrow \left. \begin{cases} 2\sqrt{x} = 18 \\ 4\sqrt{y} = 2 \end{cases} \right\} \Rightarrow \boxed{x = 81 \\ y = 16}$$

$$14. \left. \begin{cases} \log_2 xy = 5 \\ \log_{\frac{1}{2}} \frac{x}{y} = 1 \end{cases} \right\} \Rightarrow \left. \begin{cases} xy = 32 \\ \frac{x}{y} = \frac{1}{2}, \quad y = 2x \end{cases} \right\} \Rightarrow \left. \begin{cases} 2x^2 = 32 \\ |x| = 4 \end{cases} \right\} \Rightarrow \left. \begin{cases} \boxed{x = 4 \quad y = 8} \\ \boxed{x = -4 \quad y = -8} \end{cases} \right\}$$

$$15. \left. \begin{cases} 8^{2x+1} = 32 \cdot 2^{4y-1} \\ 5 \cdot 5^{x-y} = \sqrt{25^{2y+1}} \end{cases} \right\} \Rightarrow \left. \begin{cases} 2^{6x+3} = 2^{4y+4} \\ 5^{x-y} = 5^{2y} \end{cases} \right\} \Rightarrow \left. \begin{cases} 6x+3 = 4y+4 \\ x-y = 2y \end{cases} \right\} \Rightarrow \left. \begin{cases} 6x-4y = 1 \\ x = 3y \end{cases} \right\} \Rightarrow \left. \begin{cases} 14y = 1 \\ x = \frac{3}{14} \end{cases} \right\} \Rightarrow \boxed{x = \frac{3}{14} \quad y = \frac{1}{14}}$$

$$16. \quad \left. \begin{array}{l} 3^y \cdot 9^x = 81 \\ \lg(x+y)^2 - \lg x = 2 \lg 3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 3^y \cdot 3^{2x} = 3^4 \\ \lg \frac{(x+y)^2}{x} = \lg 9 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2x+y = 4 \\ (x+y)^2 = 9x \end{array} \right\} \Rightarrow (4-x)^2 = 9x \Rightarrow x^2 - 17x + 16 = 0$$

I.  $x=1 \quad y=2$

II.  $x=16 \quad y=-28$

$$17. \quad \left. \begin{array}{l} 3 \cdot 2^{x+y} - 5 \cdot 2^{x-y} = 182 \\ 5 \cdot 2^x \cdot 2^y - 4 \cdot 2^x \cdot 2^{-y} = 312 \end{array} \right\} \Rightarrow \left. \begin{array}{l} X := 2^{x+y} \\ Y := 2^{x-y} \end{array} \right\} \Rightarrow \left. \begin{array}{l} 3X - 5Y = 182 \\ 5X - 4Y = 312 \end{array} \right\} \Rightarrow \left. \begin{array}{l} X = 64 \\ Y = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2^{x+y} = 2^6 \\ 2^{x-y} = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x+y=6 \\ x-y=1 \end{array} \right\} \Rightarrow \boxed{x=3.5 \quad y=2.5}$$

$$18. \quad \left. \begin{array}{l} 4x - 5y = 7 \\ 2x + y = 7 \\ 8x - 17y = 5 \\ 3x + y = 10 \end{array} \right\} \Rightarrow$$

A 4. és 2. egyenlet különbsége:  $x=3$ ,  
majd a 2. egyenletből:  $y=7-2 \cdot 3=1$ .

A fenti  $x=3$ ,  $y=1$  értékeket az 1. egyenletbe helyettesítve az egyenlőség teljesül,  
a 3. egyenletbe helyettesítve azonban nem,

így az egyenletrendszernek nincsen megoldása.

$$19. \quad \left. \begin{array}{l} x+y = 2 \\ x-z = 3 \\ x-y+z = 2 \end{array} \right\} \Rightarrow$$

A három egyenletet összeadva kapjuk:  $3x=7 \Rightarrow x=\frac{7}{3}$ ,

majd az 1. egyenletből:  $y=2-\frac{7}{3}=-\frac{1}{3}$ , a 2. egyenletből:  $z=\frac{7}{3}-3=-\frac{2}{3}$ ,

így az egyenletrendszer megoldása:  $x=\frac{7}{3}$ ,  $y=-\frac{1}{3}$ ,  $z=-\frac{2}{3}$ .

$$20. \quad \left. \begin{array}{l} -x+2y+z = 2 \\ x+3z = -2 \\ 2x+y+z = 3 \end{array} \right\} \Rightarrow$$

Az 1. és 2. egyenlet összege:  $2y+4z=0, \Rightarrow y=-2z$ ,

Az 1. egyenlet 2-szeresének és 3. egyenletnek az összege:  $5y+3z=7$ ,

$$\Rightarrow 5 \cdot (-2z) + 3z = 7, \Rightarrow z = -1, \quad x = -2 - 3 \cdot (-1), \quad y = -2 \cdot (-1)$$

így az egyenletrendszer megoldása:  $x=1$ ,  $y=2$ ,  $z=-1$ .