Alkalmazható képletek:

$$\boxed{\int x^{\alpha}dx=\frac{x^{\alpha+1}}{\alpha+1}+C}$$ ha $\alpha\neq-1$ valós konstans

$$\boxed{\int x^{-1}dx=\ln|x|+C}$$ ha $\alpha=-1$,

$$\boxed{\int f(ax+b)dx=\frac{1}{a}F(ax+b)+C \qquad ahol \quad F'=f}$$

Tipikus példa: $\int\frac{1}{3x+4}dx=\frac{1}{3}\int\frac{3}{3x+4}dx=\frac{1}{3}\ln|3x+4|+C$

$$\boxed{\int g(f(x))f'(x)dx=G(f(x))+C \qquad ahol \quad G'=g}$$

Speciális esetei:

$$\boxed{\int\frac{f'(x)}{f(x)}dx=\ln|f(x)|+C}$$

$$\boxed{\int(f(x))^{\alpha}f'(x)dx=\frac{(f(x))^{\alpha+1}}{\alpha+1}+C}$$

$$\boxed{\int f'(x)\cdot g(x)dx=f(x)g(x)-\int f(x)\cdot g'(x)dx}$$

FELADATOK és MEGOLDÁSOK

1. $\int x^2(x^2-1)dx=\int(x^4-x^2)dx=\frac{x^5}{5}-\frac{x^3}{3}+C$ 3.

2. $\int\frac{1}{x^2}dx=\int x^{-2}dx=\frac{x^{-1}}{-1}+C=-\frac{1}{x}+C$

3. $\int\frac{\sqrt{x}-x+x^4}{x^2}dx=\int\left(x^{-\frac{3}{2}}-x^{-1}+x^2\right)dx=\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}-\ln|x|+\frac{x^3}{3}+C$

4. $\int\frac{(x+1)(x^2-3)}{3x^2}dx$ elvégezve a szorzást tagonként lehet integrálni

4./b $\int\frac{x^2+x-6}{x-2}dx=\int\frac{(x-2)(x+3)}{x-2}dx=\int(x+3)dx=\frac{x^2}{2}+3x+C$

5. $\int\frac{x^2-4x+7}{x-2}dx=\int\frac{(x-2)^2-4+7}{x-2}dx=\int\left((x-2)+\frac{3}{x-2}\right)dx=\frac{x^2}{2}-2x+\ln|x-2|+C$

6. $\int\left(2x+3\sqrt{x}+\sqrt[3]{x+1}\right)dx=x^2+3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{(x+1)^{\frac{4}{3}}}{\frac{4}{3}}+C$

7. $\int \frac{3}{2x+5}dx = \frac{3}{2}\int \frac{2}{2x+5}dx = \frac{3}{2}\ln|2x+5| + C$

8. $\int \frac{3x+10}{2x+5}dx = 3\int \frac{x+\frac{10}{3}}{2x+5}dx = \frac{3}{2}\int \frac{2x+\frac{20}{3}}{2x+5}dx = \frac{3}{2}\int \frac{2x+5-5+\frac{20}{3}}{2x+5}dx = \frac{3}{2}\int\left(1+\frac{-5+\frac{20}{3}}{2x+5}\right)dx =$

$= \frac{3}{2}\int (1+\frac{\frac{5}{3}}{2x+5})dx = \frac{3}{2}\left(\int dx + \frac{5}{3}\int\left(\frac{1}{2x+5}\right)dx\right) = \frac{3}{2}x + \frac{5}{2}\int\left(\frac{1}{2x+5}\right)dx = \frac{3}{2}x + \frac{5}{4}\int\left(\frac{2}{2x+5}\right)dx = \frac{3}{2}x + \frac{5}{4}\ln|2x+5| + C$

Az olyan $\int \frac{1}{ax^2+bx+c}dx$ alakú feladatokban ahol a nevezőben lévő másodfokú polinomnak nincs valós gyökeke, érdemes a nevezőben lévő kifejezést teljes négyzetté alakítani és $\int \frac{1}{(dx+e)^2+1}dx$ formára hozni.

Ekkor fel lehet használni, hogy $\boxed{\operatorname{arc}' tg\,(dx+e) = \frac{1}{1+(dx+e)^2}(d)}$ mint a következő feladatban:

9. $\int \frac{3}{2x^2+5}dx = 3\int \frac{1}{2x^2+5}dx = \frac{3}{5}\int \frac{1}{\frac{2}{5}x^2+1}dx = \frac{3}{5}\int \frac{1}{\left(\sqrt{\frac{2}{5}}x\right)^2+1}dx = \frac{3}{5}\sqrt{\frac{5}{2}}\int \frac{\sqrt{\frac{2}{5}}}{\left(\sqrt{\frac{2}{5}}x\right)^2+1}dx = \frac{3}{5}\sqrt{\frac{5}{2}}\arctan\left(\sqrt{\frac{2}{5}}x\right) + C$

A következő feladatban alkalmazható formulák:

$$\boxed{\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C}$$

$$\boxed{\int (f(x))^\alpha f'(x)dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$$

10. $\int \frac{3x}{2x^2+5}dx = 3\int \frac{x}{2x^2+5}dx = \frac{3}{4}\int \frac{4x}{2x^2+5}dx = \frac{3}{4}\ln(2x^2+5) + C$

11. $\int \frac{3x+10}{2x^2+5}dx = \int \frac{3x}{2x^2+5}dx + \int \frac{10}{2x^2+5}dx$, az első tag olyan mint a 9., a második pedig mint a 10. feladat

12. $\int \frac{3x^2}{2x+5}dx$ $=?$ E feladatban átalakítjuk a racionális tört függvényt egy egész polinom és egy valódi törtfüggvény összegére. Ez megoldható ügyes átalakítással vagy polinom osztással:

$\int \frac{3x^2}{2x+5}dx = \frac{3}{4}\int \frac{4x^2+10x-10x+25-25}{2x+5}dx = \frac{3}{4}\int \frac{(2x+5)^2-10x-25}{2x+5}dx = \frac{3}{4}\int\left((2x+5)+\frac{-10x-25}{2x+5}\right)dx$

$= \frac{3}{4}\int (2x+5)\,dx - \frac{3}{4}\int \frac{10x+25}{2x+5}dx = \frac{3}{4}\int (2x+5)\,dx - \frac{3}{4}\int \frac{5(2x+5)}{2x+5}dx = \frac{3}{4}\int (2x+5)\,dx - \frac{15}{4}\int dx = \frac{3}{4}x^2 + 5x + \frac{15}{4}x + C$

13. $\int \frac{3x^2}{2x^2+5}dx = 3\int \frac{x^2}{2x^2+5}dx = \frac{3}{2}\int \frac{2x^2}{2x^2+5}dx = \frac{3}{2}\int \frac{2x^2+5-5}{2x^2+5}dx = \frac{3}{2}\int\left(1+\frac{-5}{2x^2+5}\right)dx$ innen olyan mint a 9. feladat

14. $\int \frac{3}{\sqrt{2x+5}}dx = 3\int (2x+5)^{-\frac{1}{2}}dx = \frac{3}{2}\int 2\cdot(2x+5)^{-\frac{1}{2}}dx = \frac{3}{2}\frac{(2x+5)^{\frac{1}{2}}}{\frac{1}{2}}+C$

$$\boxed{\int \left(f(x)\right)^{\alpha} f'(x)dx = \frac{\left(f(x)\right)^{\alpha+1}}{\alpha+1}+C}$$

15. $\int \frac{3x}{\sqrt{2x+5}}dx = ?$ NEHÉZ!

$\sqrt{2x+5} = t$, $2x+5 = t^2$, $2x = t^2-5$, $x = \frac{t^2}{2}-\frac{5}{2}$, $\frac{dx}{dt} = t$, $dx = t\cdot dt$,

$$\int \frac{3x}{\sqrt{2x+5}}dx = \int \frac{3\frac{t^2-5}{2}}{t}\cdot t\cdot dt = 3\int \frac{t^2-5}{2}\cdot dt = \frac{1}{2}t^3 - \frac{15}{2}t + C = \frac{1}{2}\left(\sqrt{2x+5}\right)^3 - \frac{15}{2}\sqrt{2x+5}+C$$

vagy parciálisan $\boxed{\begin{array}{c} 2(2x+5)^{-\frac{1}{2}} = f'(x) \\ x = g(x) \end{array}}$

$$\boxed{\int f'(x)\cdot g(x)dx = f(x)g(x) - \int f(x)\cdot g'(x)dx}$$

16 $\int \frac{3}{\sqrt{2x^2+5}}dx = ?$ NEHÉZ! Ezt vissza kell vezetni $\boxed{\int \frac{1}{\sqrt{(bx)^2+1}}dx = \frac{1}{b}\int \frac{b}{\sqrt{(bx)^2+1}}dx = \frac{1}{b}arsh(bx)+c}$ alakúvá

$$\int \frac{3}{\sqrt{2x^2+5}}dx = 3\int \frac{1}{\sqrt{2x^2+5}}dx = 3\int \frac{1}{\sqrt{\left(\sqrt{2}x\right)^2+5}}dx = 3\int \frac{1}{\sqrt{5\left(\frac{\left(\sqrt{2}x\right)^2}{5}+1\right)}}dx = \frac{3}{\sqrt{5}}\int \frac{1}{\sqrt{\left(\left(\frac{\sqrt{2}x}{\sqrt{5}}\right)^2+1\right)}} =$$

$$= \frac{3}{\sqrt{5}}\int \frac{1}{\sqrt{\left(\left(\frac{\sqrt{2}x}{\sqrt{5}}\right)^2+1\right)}}dx = \frac{3}{\sqrt{5}}\int \frac{1}{\sqrt{\left(\left(\sqrt{\frac{2}{5}}\cdot x\right)^2+1\right)}}dx = \frac{3}{\sqrt{5}}\sqrt{\frac{5}{2}}\int \frac{\sqrt{\frac{2}{5}}}{\sqrt{\left(\left(\sqrt{\frac{2}{5}}\cdot x\right)^2+1\right)}}dx = \frac{3}{\sqrt{2}}arsh\left(\sqrt{\frac{2}{5}}\cdot x\right)+C$$

17. $\int \frac{3x}{\sqrt{2x^2+5}}dx = 3\int x\left(2x^2+5\right)^{-\frac{1}{2}}dx = \frac{3}{4}\int 4x\left(2x^2+5\right)^{-\frac{1}{2}}dx = \frac{3}{4}\frac{\left(2x^2+5\right)^{\frac{1}{2}}}{\frac{1}{2}}+C$

$$\boxed{\int \left(f(x)\right)^{\alpha} f'(x)dx = \frac{\left(f(x)\right)^{\alpha+1}}{\alpha+1}+C}$$

18. $\int \frac{3x^2+4x+7{,}5}{\sqrt{2x^2+5}}dx = \int \left(\frac{3x^2}{\sqrt{2x^2+5}}+\frac{4x}{\sqrt{2x^2+5}}+\frac{7{,}5}{\sqrt{2x^2+5}}\right)dx$ tagonként a 17, 16, 15 feladatok alapján

19. $\int (x^5+5^x)dx = \int x^5dx + \int 5^x dx = \frac{x^6}{6}+\frac{1}{\ln 5}\int \ln 5\cdot 5^x dx = \frac{x^6}{6}+\frac{5^x}{\ln 5}+C$

20. $\int (4\sin x - 3\cos x)dx = -4\cos x - 3\sin x + C$

21. $\int tgx\cdot dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{-\sin x}{\cos x}dx = -\ln|\cos x|+C$

$$\boxed{\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|+C}$$

22. $\int tg^2 x\cdot dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int \left(\frac{1}{\cos^2 x}-1\right)dx = tgx - x + C$

23. $\int \frac{\cos 2x}{\cos x - \sin x}dx = \int \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}dx = \int (\cos x + \sin x)dx = \sin x - \cos x + C$

24. $\int \frac{1}{shx+chx}dx = \int \frac{1}{\frac{e^x-e^{-x}}{2}+\frac{e^x+e^{-x}}{2}}dx = \int \frac{2}{e^x-e^{-x}+e^x+e^{-x}}dx = \int e^{-x}dx = -\int -e^{-x}dx = -e^{-x}+C$

25. $\int \sin^2 x \cdot dx = \int \frac{1-\cos 2x}{2} dx \int (\frac{1}{2} - \frac{\cos 2x}{2}) dx = \frac{1}{2}x - \frac{1}{4}\int 2\cos 2x dx = \frac{1}{2}x - \frac{\sin 2x}{4} + C$

26. $\int \frac{e^x}{e^x+11} dx = \ln\left|e^x+11\right| + C$ $\boxed{\int \frac{f'(x)}{f(x)} dx = \ln\left|f(x)\right| + C}$

27. $\int \frac{\cos 3x}{8+\sin 3x} dx = \frac{1}{3}\int \frac{3\cos 3x}{8+\sin 3x} dx = \frac{1}{3}\ln\left|8+\sin 3x\right| + C$ $\boxed{\int \frac{f'(x)}{f(x)} dx = \ln\left|f(x)\right| + C}$

28. $\int \frac{dx}{x(\ln x + 2008)} = \int \frac{\frac{1}{x}}{(\ln x + 2008)} dx = \ln\left|\ln x + 2008\right| + C$ $\boxed{\int \frac{f'(x)}{f(x)} dx = \ln\left|f(x)\right| + C}$

29. $\int x\sqrt{1-x^2}\, dx = -\frac{1}{2}\int (-2x)\sqrt{1-x^2}\, dx = -\frac{1}{2}\int (-2x)(1-x^2)^{\frac{1}{2}} dx = -\frac{1}{2}\frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

30. $\int \frac{x}{\sqrt{x^2+1}} dx = \frac{1}{2}\int (2x)(x^2+1)^{-\frac{1}{2}} dx = \frac{1}{2}\frac{(x^2+1)^{\frac{1}{2}}}{\frac{1}{2}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

31. $\int \frac{\cos x}{\sqrt{\sin x}} dx = \int (\cos x)(\sin x)^{-\frac{1}{2}} dx = \frac{(\sin x)^{\frac{1}{2}}}{\frac{1}{2}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

32. $\int \frac{\sqrt{\ln x}}{x} dx = \int \frac{1}{x}(\ln x)^{\frac{1}{2}} dx = \frac{(\ln x)^{\frac{1}{2}}}{\frac{1}{2}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

33. $\int \sqrt[3]{\cos^7 x} \cdot \sin x dx = -\int (\cos x)^{\frac{7}{3}} \cdot (-\sin x)\, dx = -\frac{(\cos x)^{\frac{10}{3}}}{\frac{10}{3}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

34. $\int e^{-x} dx = -\int (-e^{-x}) dx = -e^{-x} + C$

35. $\int 5^{2-3x} dx = \int 25 \cdot 5^{-3x} dx = 25\frac{-1}{3\ln 5}\int 5^{-3x}(-3\ln 5)\, dx = -\frac{25}{3\ln 5}5^{-3x} + C$

36. $\int e^{\sin x}\cos x dx = \int e^{\sin x}\cos x dx = e^{\sin x} + C$ $\boxed{\int g(f(x)) f'(x) dx = G(f(x)) + C, ahol \quad G' = g}$

37. $\int x\sqrt{25+x^2}\, dx = \frac{1}{2}\int (2x)(25+x^2)^{\frac{1}{2}} dx = \frac{1}{2}\frac{(25+x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

38. $\int \frac{14}{(6-4x)^7} dx = -14\frac{1}{4}\int (6-4x)^{-7}(-4) dx = -\frac{7}{2}\frac{(6-4x)^{-6}}{-6} + C$

39. $\int \frac{\ln x}{x} dx = \int (\ln x)\frac{1}{x} dx = \frac{(\ln x)^2}{2} + C$ $\boxed{\int (f(x))^\alpha f'(x) dx = \frac{(f(x))^{\alpha+1}}{\alpha+1} + C}$

A következő feladatokban használhatjuk a parciális integrálás formuláját, mely a szorzat deriválási szabályának megfordítása: $\boxed{\int f'(x)\cdot g(x)dx = f(x)g(x)-\int f(x)\cdot g'(x)dx}$

40. $\int x\sin 3x dx = -\frac{1}{3}\int x[\sin 3x(-3)]dx = -\frac{1}{3}\left[(x\cos 3x)-\int 3\sin 3x dx\right] = -\frac{1}{3}\left[(x\cos 3x)-\int \cos 3x dx\right] =$

$= -\frac{1}{3}\left[(x\cos 3x)-\frac{\sin 3x}{3}\right]+C$ $\boxed{\begin{matrix}-3\sin 3x = f'(x)\\ x = g(x)\end{matrix}}$

41. $\int x\sin x\cos x dx = \frac{1}{2}\int x\sin 2x dx = -\frac{1}{4}\int x(-2)\sin 2x dx = -\frac{1}{4}\left[x\cos 2x-\int \cos 2x dx\right] = -\frac{1}{4}\left[x\cos 2x-\frac{1}{2}\sin 2x\right]+C$

$\boxed{\begin{matrix}-2\sin 2x = f'(x)\\ x = g(x)\end{matrix}}$

42. ---

43. $\int \ln x dx = \int (1)\cdot \ln x dx = x\ln x-\int x\frac{1}{x}dx = x\ln x - x + C = x(\ln x-1)+C$ $\boxed{\begin{matrix}1 = f'(x)\\ \ln x = g(x)\end{matrix}}$

44. NEHÉZ! fel lehet használni az előző feladat eredményét és a parciális integrálás elvét.

$\int \ln^2 x dx = \int \ln x\cdot \ln x\cdot dx = \ln x\cdot x(\ln x-1)-\int x(\ln x-1)\frac{1}{x}dx = \ln x\cdot x(\ln x-1)-\int(\ln x-1)dx = \ln x\cdot x(\ln x-1)-\int(\ln x-1)dx =$

$= \ln x\cdot x(\ln x-1)-x(\ln x-1)+x+C = x(\ln x-1)^2+x+C$

vagy helyettesítéssel:

$\int \ln^2 x\cdot dx = ?,\qquad \ln x = t \Rightarrow\quad x = e^t \Rightarrow \frac{dx}{dt} = e^t \quad\Rightarrow dx = e^t\cdot dt$

$\int \ln^2 x\cdot dx = \int t^2 e^t dt = t^2e^t-\int 2t\cdot e^t dt = t^2e^t-2\left(te^t-\int e^t dt\right) = t^2e^t-2\left(te^t-e^t\right)+C = e^t(t^2-2t+2)+C = x\left(\ln^2 x-2\ln x+2\right)+C$

45. $\int x^2\ln x dx = \frac{x^3}{3}\ln x-\int \frac{x^3}{3}\cdot\frac{1}{x}dx = \frac{x^3}{3}\ln x-\int \frac{x^2}{3}\cdot dx = \frac{x^3}{3}\ln x-\frac{x^3}{9}+C$ $\boxed{\begin{matrix}x^2 = f'(x)\\ \ln x = g(x)\end{matrix}}$

46. $\int x^3e^{-x^2}dx = -\frac{1}{2}\int x^2\left(-2xe^{-x^2}\right)dx = -\frac{1}{2}\left[x^2e^{-x^2}-\int 2xe^{-x^2}dx\right] = -\frac{1}{2}\left[x^2e^{-x^2}+\int(-2x)e^{-x^2}dx\right] =$

$= -\frac{1}{2}\left[x^2e^{-x^2}+\int(-2x)e^{-x^2}dx\right] = -\frac{1}{2}\left[x^2e^{-x^2}+e^{-x^2}\right]+C = -\frac{1}{2}e^{-x^2}\left[x^2+1\right]+C$

47. $\int arctgx dx = \int (1)\cdot arctgx dx = xarctgx-\int x\frac{1}{1+x^2}dx = xarctgx-\frac{1}{2}\int\frac{2x}{1+x^2}dx = xarctgx-\frac{1}{2}\ln(1+x^2)+C$

$\boxed{\begin{matrix}1 = f'(x)\\ arctgx = g(x)\end{matrix}}$

48. $\int x\cdot arctgx dx = \frac{x^2}{2}arctgx-\frac{1}{2}\int\frac{x^2}{1+x^2}dx = \frac{x^2}{2}arctgx-\frac{1}{2}\int\frac{x^2+1-1}{1+x^2}dx = \frac{x^2}{2}arctgx-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx =$

$= \frac{x^2}{2}arctgx-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx = \frac{x^2}{2}arctgx-\frac{1}{2}(x-arctgx)+C$ $\boxed{\begin{matrix}x = f'(x)\\ arctgx = g(x)\end{matrix}}$