Anna Rudas:
Asymptotic behaviour of random growing trees
SUMMARY

In my PhD thesis I investigate the asymptotic properties of a random tree growth model which generalizes the basic concept of preferential attachment.

In this family of models, the tree stems from a root in the beginning, and vertices are added one at a time, the new vertex always attaching to exactly one already existing vertex. The rule by which the new vertex chooses its "parent", is dependent on the degree distribution apparent in the tree at the time the vertex is born. The dependence on the degree structure is characterised by a *weight function* $w : \mathbb{N} \to \mathbb{R}_+$, the function being the parameter of the model. The tree version of the well-known Barabási–Albert graph model corresponds to a special case, namely, when w is chosen to be linear.

Our main *local results*, stated in Theorem 3.2 in the thesis, focus on the neighborhood of the *typical* vertex, as follows.

- We determine the asymptotic distribution of the degree sequence, which equivalently gives the limiting distribution of the degree of a (uniformly) randomly selected vertex.
- We also give the asymptotic distribution of the subtree under a randomly selected vertex.
- Moreover, we present the asymptotic distribution of the whole tree, seen from a randomly selected vertex.

Global properties capture phenomena observable by looking at the whole tree in the limit. The concept of the "limiting success level" of a fixed vertex, leads us to the definition of a certain random measure μ on the leaves of the limiting tree, which captures a global property of the tree growth.

We prove the following results about μ.

- The limiting entropies (as time tends to infinity) of the random measures on the different generations converge to a constant with probability one, as we let the generation level to infinity. This constant h is called the entropy of the limiting measure μ. This result is stated in Theorem 4.1 of the thesis.
- The Hausdorff and the packing dimension of the random limiting measure μ are constant and equal with probability one. The entropy and the dimension satisfy the usual simple relation $dimension = \frac{entropy}{Ljapunov\, exponent}$. Moreover, the local dimension of μ equals the Hausdorff dimension at μ-almost every point. This result is stated in Theorem 4.2 of the thesis.
- Given the *weight function* w, we provide an explicit formula for the entropy, and thus for the Hausdorff dimension, in terms of w. This constitutes Theorem 4.3 in the thesis.