

## 1. Elemi algebrai azonosságok

Műveletek hatványokkal: tegyük fel, hogy  $a, b \in \mathbb{R}^+$ ,  $\alpha, \beta \in \mathbb{N}^+$ , ekkor

$$\begin{array}{l|l|l} a^\alpha \cdot b^\alpha = (a \cdot b)^\alpha & a^\alpha \cdot a^\beta = a^{\alpha+\beta} & (a^\alpha)^\beta = a^{\alpha \cdot \beta} \\ a^{-\alpha} = \frac{1}{a^\alpha} & \frac{a^\alpha}{a^\beta} = a^{\alpha-\beta} & a^0 = 1 \\ \sqrt[\alpha]{a} = a^{\frac{1}{\alpha}} & \sqrt[\alpha]{a^\beta} = a^{\frac{\beta}{\alpha}} = (\sqrt[\alpha]{a})^\beta & \sqrt[\alpha]{a \cdot b} = \sqrt[\alpha]{a} \cdot \sqrt[\alpha]{b} \end{array}$$

## 2. Műveletek logaritmussal

Tegyük fel, hogy  $a \in \mathbb{R}^+ \setminus \{1\}$ ,  $b, c, d \in \mathbb{R}^+$ ,  $\alpha \in \mathbb{R}$  ekkor definíció szerint  $\log_a b = c \Leftrightarrow a^c = b$ , és

$$\begin{array}{l|l} \log_a(c \cdot d) = \log_a c + \log_a d & \log_a(b^\alpha) = \alpha \log_a b \\ \log_a\left(\frac{b}{c}\right) = \log_a b - \log_a c & \log_a 1 = 0, \log_a a = 1 \end{array}$$

## 3. Trigonometrikus azonosságok

Tegyük fel, hogy  $\alpha, \beta \in \mathbb{R}$ , ekkor  $\sin^2 \alpha + \cos^2 \alpha = 1$ , továbbá

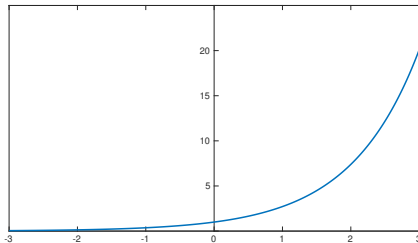
$$\begin{array}{l|l} \sin(-\alpha) = -\sin \alpha & \cos(-\alpha) = \cos(\alpha) \\ \sin(\alpha + \pi) = -\sin(\alpha) & \cos(\alpha + \pi) = -\cos \alpha \\ \sin\left(\alpha + \frac{\pi}{2}\right) = \cos \alpha & \cos\left(\alpha + \frac{\pi}{2}\right) = -\sin \alpha \\ \sin(2\alpha) = 2 \sin \alpha \cos \alpha & \cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha \\ \sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} & \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2} \\ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha & \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha & \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \end{array}$$

## 4. Nevezetes szögek szögfüggvényei

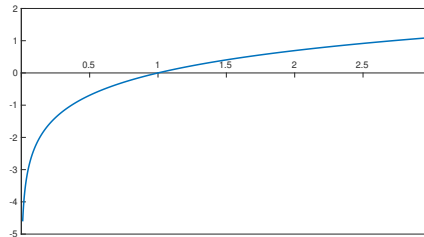
Tegyük fel, hogy  $k \in \mathbb{Z}$ , ekkor

|   |  |   |
|---|--|---|
| $\sin \frac{\pi}{6} = \frac{1}{2}$                | $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$    | $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$         |
| $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$         | $\cos \frac{\pi}{3} = \frac{1}{2}$           | $\tan \frac{\pi}{3} = \sqrt{3}$                   |
| $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$         | $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$    | $\tan \frac{\pi}{4} = 1$                          |
| $\sin \left(\frac{\pi}{2} + k\pi\right) = (-1)^k$ | $\cos \left(\frac{\pi}{2} + k\pi\right) = 0$ | $\tan \left(\frac{\pi}{2} + k\pi\right)$ nem ért. |
| $\sin(k\pi) = 0$                                  | $\cos(k\pi) = (-1)^k$                        | $\tan(k\pi) = 0$                                  |

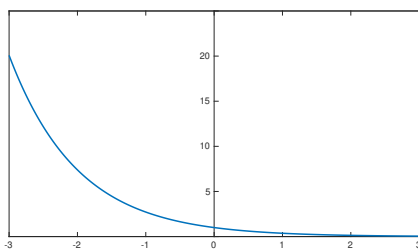
## 5. Elemi függvények



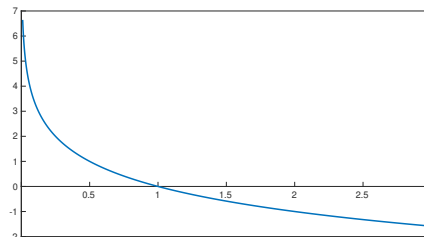
$f(x) = a^x, a > 1$   
 $D_f = \mathbb{R}$   
 $R_f = \mathbb{R}^+$   
 $\lim_{x \rightarrow \infty} a^x = \infty$   
 $\lim_{x \rightarrow -\infty} a^x = 0$   
 szig. mon. nő, folytonos  $\mathbb{R}$ -en  
 inverze :  $\log_a x$   
 $(a^x)' = \ln(a) a^x$



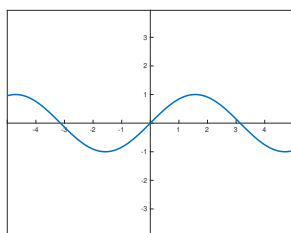
$f(x) = \log_a(x), a > 1$   
 $D_f = \mathbb{R}^+$   
 $R_f = \mathbb{R}$   
 $\lim_{x \rightarrow \infty} \log_a(x) = \infty$   
 $\lim_{x \rightarrow 0^+} \log_a(x) = -\infty$   
 szig. mon. nő, folytonos  $\mathbb{R}^+$ -en  
 inverze :  $a^x$   
 $(\log_a x)' = \frac{1}{\ln(a) \cdot x}$



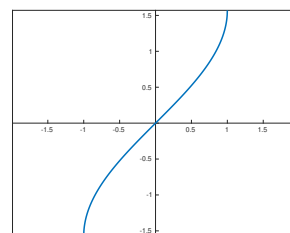
$f(x) = a^x, 0 < a < 1$   
 $D_f = \mathbb{R}$   
 $R_f = \mathbb{R}^+$   
 $\lim_{x \rightarrow \infty} a^x = 0$   
 $\lim_{x \rightarrow -\infty} a^x = \infty$   
 szig. mon. csökk., folytonos  $\mathbb{R}$ -en  
 inverze :  $\log_a x$   
 $(a^x)' = \ln(a) a^x$



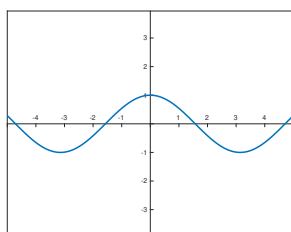
$f(x) = \log_a(x), 0 < a < 1$   
 $D_f = \mathbb{R}^+$   
 $R_f = \mathbb{R}$   
 $\lim_{x \rightarrow \infty} \log_a(x) = -\infty$   
 $\lim_{x \rightarrow 0^+} \log_a(x) = \infty$   
 szig. mon. csökk., folytonos  $\mathbb{R}^+$ -en  
 inverze :  $a^x$   
 $(\log_a x)' = \frac{1}{\ln(a) \cdot x}$



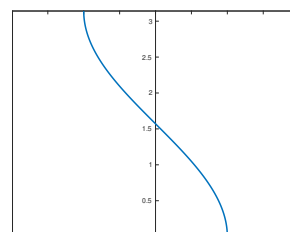
$f(x) = \sin(x)$ ,  
 $D_f = \mathbb{R}$   
 $R_f = [-1, 1]$   
 folytonos  $\mathbb{R}$ -en, páratlan,  $2\pi$ -periódikus  
 inverze :  $\arcsin(x)$   
 $(\sin x)' = \cos(x)$



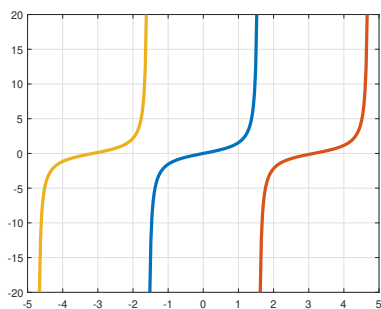
$f(x) = \arcsin(x)$   
 $D_f = [-1, 1]$   
 $R_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$   
 szig. mon. nő, folytonos  $[-1, 1]$ -en, páratlan  
 inverze :  $\sin(x)$   
 $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$



$f(x) = \cos(x)$ ,  
 $D_f = \mathbb{R}$   
 $R_f = [-1, 1]$   
 folytonos  $\mathbb{R}$ -en, páros,  $2\pi$ -periódikus  
 inverze :  $\arccos(x)$   
 $(\cos x)' = -\sin(x)$

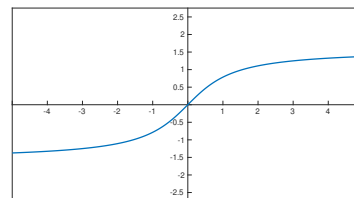


$f(x) = \arccos(x)$   
 $D_f = [-1, 1]$   
 $R_f = [0, \pi]$   
 szig. mon. csökken, folytonos  $[-1, 1]$ -en  
 inverze :  $\cos(x)$   
 $(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$



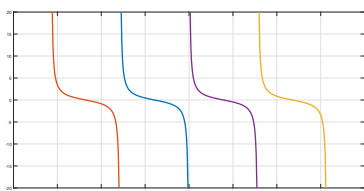
$f(x) = \tan(x)$ ,  
 $D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$   
 $R_f = \mathbb{R}$   
 folytonos  $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$ -en, páratlan,  
 $\pi$ -periódikus

inverze :  $\arctan(x)$   
 $(\tan x)' = 1 + \tan^2(x) = \frac{1}{\cos^2(x)}$



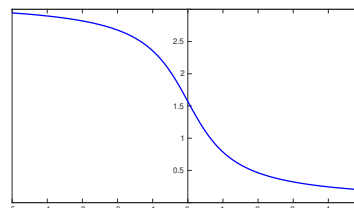
$f(x) = \arctan(x)$   
 $D_f = \mathbb{R}$   
 $R_f = (-\frac{\pi}{2}, \frac{\pi}{2})$   
 szig. mon. nő, folytonos  $\mathbb{R}$ -en, páratlan

$\lim_{x \rightarrow \infty} \arctan x = \frac{\pi}{2}$ ,  $\lim_{x \rightarrow -\infty} \arctan x = -\frac{\pi}{2}$   
 inverze :  $\tan(x)$   
 $(\arctan x)' = \frac{1}{1+x^2}$

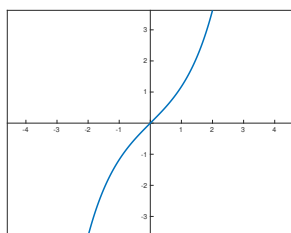


$f(x) = \cotan(x)$ ,  
 $D_f = \mathbb{R} \setminus \{k\pi\}$   
 $R_f = \mathbb{R}$   
 folytonos  $\mathbb{R} \setminus \{k\pi\}$ -en, páratlan,  $\pi$ -periódikus

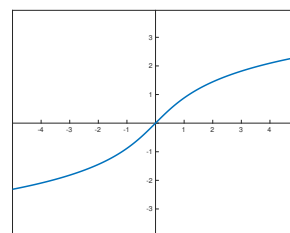
inverze :  $\operatorname{arccotan}(x)$   
 $(\cotan x)' = -1 - \cotan^2(x) = -\frac{1}{\sin^2(x)}$



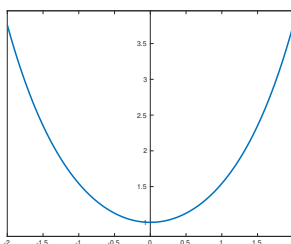
$f(x) = \operatorname{arccotan}(x)$   
 $D_f = \mathbb{R}$   
 $R_f = (0, \pi)$   
 szig. mon. csökken, folytonos  $\mathbb{R}$ -en  
 $\lim_{x \rightarrow \infty} \operatorname{arccotan} x = 0$ ,  $\lim_{x \rightarrow -\infty} \operatorname{arccotan} x = \pi$   
 inverze :  $\cotan(x)$   
 $(\operatorname{arccotan} x)' = \frac{-1}{1+x^2}$



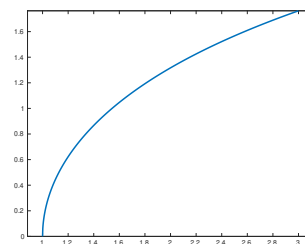
$f(x) = \sinh(x)$ ,  
 $D_f = \mathbb{R}$   
 $R_f = \mathbb{R}$   
 $\lim_{x \rightarrow \infty} \sinh x = \infty$   
 $\lim_{x \rightarrow -\infty} \sinh x = -\infty$   
 szig. mon. nő, folytonos  $\mathbb{R}$ -en, páratlan  
 inverze :  $\operatorname{arsinh}(x)$   
 $(\sinh x)' = \cosh(x)$



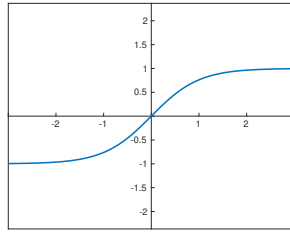
$f(x) = \operatorname{arsinh}(x)$   
 $D_f = \mathbb{R}$   
 $R_f = \mathbb{R}$   
 $\lim_{x \rightarrow \infty} \operatorname{arsinh} x = \infty$   
 $\lim_{x \rightarrow -\infty} \operatorname{arsinh} x = -\infty$   
 szig. mon. nő, folytonos  $\mathbb{R}$ -en, páratlan  
 inverze :  $\sinh(x)$   
 $(\operatorname{arsinh} x)' = \frac{1}{\sqrt{x^2+1}}$



$f(x) = \cosh(x)$ ,  
 $D_f = \mathbb{R}$   
 $R_f = [1, \infty)$   
 $\lim_{x \rightarrow \infty} \cosh x = \infty$   
 $\lim_{x \rightarrow -\infty} \cosh x = \infty$   
 folytonos  $\mathbb{R}$ -en, páros  
 inverze :  $\operatorname{arcosh}(x)$   
 $(\cosh x)' = \sinh(x)$



$f(x) = \operatorname{arcosh}(x)$   
 $D_f = [1, \infty)$   
 $R_f = \mathbb{R}^+$   
 $\lim_{x \rightarrow \infty} \operatorname{arcosh} x = \infty$   
 szig. mon. nő, folytonos  $[1, \infty)$ -en  
 inverze :  $\cosh(x)$   
 $(\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}$



$$f(x) = \tanh(x) = \frac{\sinh(x)}{\cosh(x)},$$

$$D_f = \mathbb{R}$$

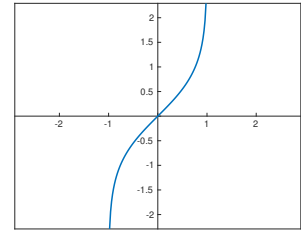
$$R_f = (-1, 1)$$

folytonos  $\mathbb{R}$ -en, páratlan, szig. mon. nő

$$\lim_{x \rightarrow \infty} \tanh x = 1, \lim_{x \rightarrow -\infty} \tanh x = -1$$

inverze :  $\operatorname{artanh}(x)$

$$(\tanh x)' = 1 - \tanh^2(x) = \frac{1}{\cosh^2(x)}$$



$$f(x) = \operatorname{artanh}(x)$$

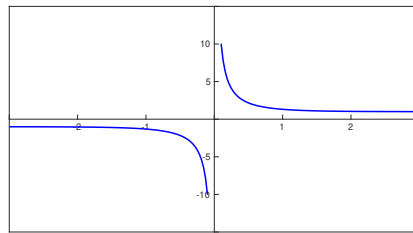
$$D_f = (-1, 1)$$

$$R_f = \mathbb{R}$$

szig. mon. nő, folytonos  $(-1, 1)$ -en, páratlan

inverze :  $\tanh(x)$

$$(\operatorname{artanh} x)' = \frac{1}{1-x^2}$$



$$f(x) = \operatorname{cotanh}(x),$$

$$D_f = \mathbb{R} \setminus \{0\}$$

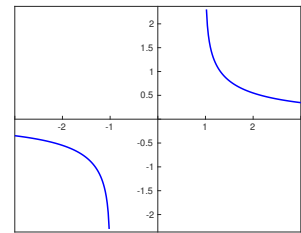
$$R_f = (-\infty, -1) \cup (1, \infty)$$

folytonos  $\mathbb{R} \setminus \{0\}$ -en, páratlan

$$\lim_{x \rightarrow \infty} \operatorname{cotanh} x = 1, \lim_{x \rightarrow -\infty} \operatorname{cotanh} x = -1$$

inverze :  $\operatorname{arcotanh}(x)$

$$(\operatorname{cotanh} x)' = 1 - \operatorname{cotanh}^2 x = -\frac{1}{\sinh^2(x)}$$



$$f(x) = \operatorname{arcotanh}(x)$$

$$D_f = (-\infty, -1) \cup (1, \infty)$$

$$R_f = \mathbb{R} \setminus \{0\}$$

páratlan, folytonos  $\mathbb{R} \setminus [-1, 1]$ -en

inverze :  $\operatorname{cotanh}(x)$

$$(\operatorname{arcotanh} x)' = \frac{1}{1-x^2}$$