We conclude our paper by a simple theorem showing that Read's attempt to solve the Liar by revising Tarski's truth schema was, in principle, bound to fail: no alternative truth schema can lead to the solution of the Liar paradox since the paradox does not depend on the *definition* of truth. More precisely, it does not depend directly on the definition of truth itself but instead on some properties of truth that can be inferred from this definition. No matter how the notion of truth is defined, as long as it can be considered as a concept of truth at all (that is, if it complies with our most basic common sense principles concerning such a notion), it leads to a contradiction. In fact, in any system in which our most fundamental logical assumptions concerning the behavior of equivalence, negation and truth hold and (some equivalents of) a sentence asserting its own non-truth can be formulated, a contradictory statement can also be derived.

The theorem itself covers the cases of those logical systems in which the Liar sentence can only be formulated implicitly (e.g. through some kind of coding such as a Gödel numbering), while its corollary is about those systems, like our informal common sense one based on natural language, where both the truth predicate and the self-reference is directly available.[24] The intended meaning of the functions T, $\ulcorner\,\urcorner$, $\neg$ and the relation $\leftrightarrow$ should be clear from the context. They represent the truth predicate, a naming function, the negation, and any kind of equivalence, respectively.

Theorem 2. Let $\mathcal{L} = (S, \leftrightarrow, \neg, \mathrm{T}, N, \ulcorner\,\urcorner)$ be a system such that $\neg$ is a unary function mapping the non-empty set S (of sentences) to itself, T is a unary function mapping the set N (of names) to S, and $\ulcorner\,\urcorner$ is a unary function mapping S onto N in a one-one way. Further, let $\leftrightarrow$ be a binary relation on S such that, for any $\sigma, \tau, \rho \in S$,

(a) $\sigma \leftrightarrow \tau$ and $\tau \leftrightarrow \rho$ imply $\sigma \leftrightarrow \rho$
(b) $\sigma \leftrightarrow \tau$ implies $\neg\sigma \leftrightarrow \neg\tau$.

Assume that, for any $\sigma, \tau \in S$,

(c) $\mathrm{T}(\ulcorner\neg\sigma\urcorner) \leftrightarrow \neg\mathrm{T}(\ulcorner\sigma\urcorner)$
(d) $\mathrm{T}(\ulcorner\mathrm{T}(\ulcorner\sigma\urcorner)\urcorner) \leftrightarrow \mathrm{T}(\ulcorner\sigma\urcorner)$
(e) $\sigma \leftrightarrow \tau$ implies $\mathrm{T}(\ulcorner\sigma\urcorner) \leftrightarrow \mathrm{T}(\ulcorner\tau\urcorner)$.

Suppose that there is a sentence Λ such that

$$\Lambda \leftrightarrow \neg\mathrm{T}(\ulcorner\Lambda\urcorner).$$

[24] Indeed, though various common language formulations of the Liar were subject to heavy criticism, there are, in fact, absolutely unobjectionable ways to achieve self-reference, and hence to formulate the paradox, in terms of natural languages, cf. [12].

Then

$$\mathrm{T}(\ulcorner\Lambda\urcorner) \longleftrightarrow \neg\mathrm{T}(\ulcorner\Lambda\urcorner).$$

Proof.

(1)	$\mathrm{T}(\ulcorner\Lambda\urcorner) \longleftrightarrow \mathrm{T}(\ulcorner\neg\mathrm{T}(\ulcorner\Lambda\urcorner)\urcorner)$	Definition of Λ, (e)
(2)	$\mathrm{T}(\ulcorner\neg\mathrm{T}(\ulcorner\Lambda\urcorner)\urcorner) \longleftrightarrow \neg\mathrm{T}(\ulcorner\mathrm{T}(\ulcorner\Lambda\urcorner)\urcorner)$	(c)
(3)	$\neg\mathrm{T}(\ulcorner\mathrm{T}(\ulcorner\Lambda\urcorner)\urcorner) \longleftrightarrow \neg\mathrm{T}(\ulcorner\Lambda\urcorner)$	(d), (b)
(4)	$\mathrm{T}(\ulcorner\Lambda\urcorner) \longleftrightarrow \neg\mathrm{T}(\ulcorner\Lambda\urcorner)$	(1),(2),(3),(a)

Corollary. Assume that the conditions of the preceding theorem hold and, further, for any $\sigma, \tau \in S$, $\sigma = \tau$ implies $\sigma \longleftrightarrow \tau$. Suppose that there is a sentence Λ such that

$$\Lambda = \neg\mathrm{T}(\ulcorner\Lambda\urcorner).$$

Then

$$\mathrm{T}(\ulcorner\Lambda\urcorner) \longleftrightarrow \neg\mathrm{T}(\ulcorner\Lambda\urcorner).$$

The theorem above is a possible abstract generalization of Tarski's theorem on the undefinability of arithmetical truth. It shows that if we liked to retain the basic characterizing properties of equivalence, negation, and truth, then we should give up the common sense requirement of the possibility of talking, without any further restriction, about the truth of sentences. In fact, this result explains the remarkable phenomenon that, in spite of the innumerable attempts to solve it since its invention two and a half millennia ago, the Liar paradox has not yet been solved. The reason is simple enough. It cannot, in principle, be solved. As our theorem shows, if we permit talking about the truth of our sentences in an unconstrained way, then some characterizing properties of truth simply lead to a contradiction. Truth and the Liar are two sides of the same coin. You can only eliminate the contradiction by giving up at least one of our most basic intuitive requirements concerning the logic of truth. The answer to the question "Which is the basic logical law that we can do without?" is, of course, a matter of taste. But *de gustibus non est disputandum.* Therefore, there is no way out of the dilemma that everybody would accept; there exists no universally acceptable solution to the Liar paradox. Clearly, tastes cannot be the subject of rational philosophical or logical discussion. In the light of this theorem and its corollary, therefore, it is entirely clear that any attempt to solve the Liar paradox is tantamount to an argument trying to convince the community of logicians that one (or more) of our basic logical notions such as truth, equivalence, or negation can be modified in such a way that, on the one hand, the modification leads to the elimination of the paradox, on the other, the most important logical laws are affected only to such an extent that can be considered a reasonable price for our getting rid of the paradox. An obvious example of such a partial revision of our logic that has proved to be completely successful is the epoch-making classic theory of truth worked out by Tarski.

To put it briefly, the theorem and its corollary show that what Read would like to do, that is, trying to eliminate the Liar paradox without giving up at least one of the fundamental intuitive requirements concerning our basic logical notions is completely hopeless. In order to solve the paradox, it is not enough to revise the definition of truth, our whole logic needs a revision. Such a revision, however, would be an entirely different and a much greater endeavour.

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