

## Required material for the theoretical part of the exam

### File A

- (1) Definitions from slides 18-20,
- (2) Chapman-Kolmogorov equation slide 44,
- (3) Definitions from slides 51-53,
- (4) everything: 87-108,
- (5) everything: 139-146.

### File B

- (1) everything: 2-5
- (2) Doubly stochastic MC: 18
- (3) recurrence of SSRW 24
- (4) detailed balance condition, time reversal 27-37
- (5) birth and death process 38-43
- (6) gambler's ruin fair and unfair cases 61-67
- (7) everything: 99-120
- (8) generator function and branching processes 135-148
- (9) everything: 151-175

### File C Everything from File C

### File D

- (1) everything 3-53
- (2) Birth and Death Chains 57-59
- (3)  $M/M/s$  queuing with balking, pure birth processes 65-72
- (4) Kolmogorov's Three-Series Theorem 73
- (5) everything 79-92
- (6)  $M/M/1$  queuing again and with finite waiting room 99- 109
- (7) Barber shop last time 110-112

### File E

- (1) Review slides: 3-8
- (2) Lemma 2.4 slide 16
- (3) Some measure theory: 17-20
- (4) Properties of Conditional expectation 21-22
- (5) Conditional expectation for  $\sigma$ -algebras slide 23
- (6) Review of measure theory slides 31-35
- (7) Conditional expectation for  $\sigma$ -algebras slides 37-47
- (8) Martingales: slides 49-55
- (9) Some further properties of martingales slides 58-64
- (10) Doubling strategy and related comments (process  $W_n$ ) slides 65-70
- (11) Stopping times slides 71- 74
- (12) Theorem 5.22 and  $M_{n \wedge T}$  slides 78-80

- (13) Exit distributions slides, Doob's optional stopping time theorem 80-88
- (14) Wald equality slide 89
- (15) Convergence theorem for non-negative supermartingales: slides 90-95
- (16) Polya Urn slides 96-101

**File F**

- (1) Review of the properties of the normal distribution on  $\mathbb{R}$ : slides 3-7
- (2) Definition of the one-dimensional Brownian motion: slide 10-11
- (3) Construction of the one-dimensional Brownian motion: slides 12-20
- (4) Equivalent definition of the Brownian motion: slide 21
- (5) Basic properties of the one-dimensional Brownian motion: slides 22-24
- (6) Brownian motion in  $\mathbb{R}^n$  and Hölder property slides 25-28.

**File G**

- (1) Definition of Fractal Percolation Cantor set. slides 3-19
- (2) Corollary 1.1 slide 20
- (3) Definition of the Hausdorff dimension slide 23
- (4) Lemma 2.2 slide 25-27
- (5) Dimension formula: proof of the upper bound
- (6) Dimension formula: proof of the lower bound
- (7) Percolation phenomenon and the Chayes, Chayes Durrett Theorem slides 41-42
- (8) Dekking Pakes Theorem slide 46
- (9) Part of the proof of Chayes, Chayes Durrett Theorem if  $M \geq 3$ .