

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\sin x \sin y = -\frac{1}{2} [\cos(x+y) - \cos(x-y)]$$

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}, \quad \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

### deriváltak:

$$(\sinh x)' = \cosh x$$

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$$(\log_a x)' = \frac{1}{x \ln a}$$

$$(x^\alpha)' = \alpha x^{\alpha-1}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln(a)$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$(\cot x)' = -\frac{1}{\sin^2 x}$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcsinh} x)' = \frac{1}{\sqrt{1+x^2}}$$

$$(\operatorname{arcosh} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\operatorname{artanh} x)' = \frac{1}{1-x^2}$$

$$(\operatorname{arcoth} x)' = \frac{1}{1-x^2}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

### differenciálási szabályok:

$$(cu)' = cu' \quad (c \text{ konstans})$$

$$(u+v)' = u' + v'$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$\frac{d}{dx} f(g(x)) = \frac{df}{dg} \frac{dg}{dx}$$

### integrálási szabályok:

$$\int cf \, dx = c \int f \, dx \quad (c \text{ konstans})$$

$$\int (f+g) \, dx = \int f \, dx + \int g \, dx$$

$$\int f(ax+b) \, dx = \frac{1}{a} F(ax+b) + c,$$

ahol  $F$  az  $f$  primitív függvénye

$$\int f(g(x))g'(x) \, dx = F(g(x)) + c,$$

ahol  $F$  az  $f$  primitív függvénye

$$\int f^\alpha f' \, dx = \frac{f^{\alpha+1}}{\alpha+1} + c, \text{ ha } \alpha \neq -1$$

$$\int \frac{f'}{f} \, dx = \ln |f| + c$$

$$\int uv' \, dx = uv - \int u'v \, dx$$

### nevezetes helyettesítések:

$$R(e^x) \quad e^x = t$$

$$R(\sqrt{ax+b}) \quad \sqrt{ax+b} = t$$

$$R\left(\frac{\sqrt{ax+b}}{\sqrt{cx+d}}\right) \quad \frac{\sqrt{ax+b}}{\sqrt{cx+d}} = t$$

$$R(\sin x, \cos x) \quad \sin x, \cos x, \tan x, \tan \frac{x}{2} = t$$

$$R(x, \sqrt{a^2 - x^2}) \quad x = a \sin t, \quad x = a \cos t$$

$$R(x, \sqrt{a^2 + x^2}) \quad x = a \sinh t$$

$$R(x, \sqrt{x^2 - a^2}) \quad x = a \cosh t$$

### integrálok:

$$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1} + c \quad (\alpha \neq -1)$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + c$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \frac{1}{\cos^2 x} \, dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} \, dx = -\cot x + c$$

$$\int \frac{1}{x} \, dx = \ln |x| + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \operatorname{arcsinh} \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \operatorname{arcosh} \frac{x}{a} + c$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \operatorname{artanh} \frac{x}{a} + c, \quad \text{ha } \left|\frac{x}{a}\right| < 1$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{a} \operatorname{arcoth} \frac{x}{a} + c, \quad \text{ha } \left|\frac{x}{a}\right| > 1$$

$$\int \tan x \, dx = -\ln |\cos x| + c$$

$$\int \cot x \, dx = \ln |\sin x| + c$$