Corollary 7.7.

Uniform graphs with a given degree sequence & $CM_n(d)$.

$\underline{d}$ $\{d_i\}_{i=1}^n$ satisfies A1 & A4. l_n even.

Then, an even $\mathcal{E}_n$ occurs w.h.p for uniform simple graph with $\underline{d}$ when it occurs w.h.p. for $CM_n(d)$.

Proof $\mathbb{P}'(\mathcal{E}_n^c) = \underset{CM}{\mathbb{P}}(\mathcal{E}_n^c / \text{Simple}) = \dfrac{\underset{CM}{\mathbb{P}}(\mathcal{E}_n^c \cap \text{Simple})}{\mathbb{P}(\text{Simple})} \leq$

$$\leq \frac{\underset{CM}{\mathbb{P}}(\mathcal{E}_n^c) \longrightarrow 0}{\mathbb{P}(\text{Simple})} = 0$$

if strictly $\oplus$

Configuration Model with iid degrees

$\{D_i\}_{i=1}^n$ D_i iid; $D_n^{(n)} = D_n + \mathbb{1}\left(\sum_{i=1}^n D_i \text{ odd}\right)$

$L_n = \sum_{i=1}^n D_i + \mathbb{1}(\sum D_i \text{ odd})$ (modify the last vertex by adding 1 stub if necessary)

Double randomness: degrees + pairing the stubs, given the degrees.

$P_k^{(n)} = \frac{1}{n} \sum_{i=1}^n \mathbb{1}(D_i^{(n)} = k)$ empirical distribution of the degrees: iid sequence

$\Rightarrow$ SLLN: $\circledast$ $P_k^{(n)} \xrightarrow{a.s.} p_k = \mathbb{P}(D_1 = k)$

furthermore $d_{TV}(P^{(n)}, p) = \sum_{k=1}^{\infty} |P_k^{(n)} - p_k| \xrightarrow{a.s.} 0$ (prove in general from $\circledast$)

Theorem 7.10. Probability of simplicity in $CM_n(D)$.

$(D_i)_{i=1}^n$ iid, $Var(D) < \infty$ $\mathbb{P}(D \geq 1) = 1$ (no isolated vertices)

$\mathbb{P}(CM_n(D) \text{ simple}) \to e^{-\frac{\nu}{2} - \frac{\nu^2}{4}}$ where $\nu = \dfrac{\mathbb{E}(D(D-1))}{\mathbb{E}D}$

Proof

1. $\frac{1}{n} L_n = \frac{1}{n} \sum D_i \xrightarrow{a.s.} \mathbb{E}D \geq 1$

$\frac{1}{n} \sum_{i=1}^n D_i(D_i - 1) \xrightarrow{a.s.} \mathbb{E}(D(D-1))$. $\Rightarrow$ A4 is satisfied with prob. 1.

$\Rightarrow$ 7.10. is a result of Thm. 7.3