DIFFERENTIAL GEOMETRY MID-TERM RE-TAKE, MAY 24, 2022

In all parts, we consider the plane curve γ defined for $0 < \varphi < \pi$ by

$$x(\varphi) = 6\cos\varphi - 4\cos^3\varphi, \quad y(\varphi) = 4\sin^3\varphi.$$

Hint: you may use the triple-angle formulae

$$\cos(3\varphi) = 4\cos^3\varphi - 3\cos\varphi, \quad \sin(3\varphi) = 3\sin\varphi - 4\sin^3\varphi.$$

(1) Compute the length of γ. (10 points)

(2) Show that the curvature of γ is $\frac{1}{3\sin\varphi}$. (20 points)

(3) Show that the evolute of γ is similar to (a reparameterization of) γ and determine the similarity transformation bringing one to the other. (20 points)