Exercise sheet 2: symmetries of the plane and of the space

Introduction to Geometry, Szilárd Szabó, special thanks to Zoltán Kovács

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Exercise 1. Let ABC be any negatively oriented triangle and AB_1C, BCA_1 and be AC_1B be postively oriented regular triangles. Denote by O the center of AC_1B. Show that then A_1OB_1 is an isosceles triangle with base angles equal to $\pi/6$.

Exercise 2. Let ℓ_1, ℓ_2, ℓ_3 be three lines in the plane and R_1, R_2, R_3 respectively the reflections with respect to them. Show that $(R_1R_2R_3)^2$ is a translation.

Exercise 3. Let $ABCDE$ be vertices of a positively oriented regular pentagon. Show that D is the center of the positively oriented regular decagon (i.e., 10-gon) with vertices A and B.

Exercise 4. Let $n = 2k + 1$ be any odd number bigger than 3. Let $P_0, \ldots, P_{n-1}$ denote consecutive vertices of a regular n-gon, increasingly labelled in the positive direction. Let N denote the intersection point of the diagonals P_0P_2 and P_1P_{k+1}. Show that the length of the segment P_0N is equal to the side of the n-gon.