NOTE TO THE PROOF OF FUNDAMENTAL THEOREM: THE CONSTANTS.

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Reminder:

- ε_1 - size of boundary layer (can be fixed arbitrarily in $(0,1)$);
- ε_2 - smallness of angles $\angle(S_{-n}, e^s)$ and of $\angle(S_n, e^u)$for $n \geq n_0$;
- ε_3 - badness parameter in bad parallelograms of type (ii) **N. B.** for singular hyperbolic algebraic automorphism of torus it can be chosen just $\varepsilon_3 = \varepsilon_1$;
- $F(\delta) : (0, \delta_0) \to \mathbb{Z}_+$ fixed function, s. t. $\lim_{\delta \to 0} F(\delta) = \infty$, so as the total measure of δ-neighborhoods of double singularities up to order $F(\delta)$ be of $o(\delta)$.

Task: bound the total measure of bad parallelograms of type (ii) (intersecting exactly one singularity of order $n_0 \leq j \leq F(\delta)$ and, moreover, such that for at least one s-side E^s one has

$$\mu(G_i^\delta \cap [E^s]^{[\varepsilon_1 \delta]} \cap U_{ic}) \leq \frac{\varepsilon_1}{4} \mu(G_i^\delta) \tag{1}$$

Claim: *if G_i^δ intersects at most one regular piece of singularity $\mathcal{R}$ of $T^{F(\delta)}$ and for a $y \in G_i^\delta$ $\gamma^s_{\text{loc}}(y)$ does not intersect G_i^δ correctly, then necessarily*

$$y \in G_i^\delta \cap \mathcal{R}^{[\varepsilon_2 \delta]} \cup U_\omega^b \tag{2}$$

The claim is straightforward.

Take a G_i^δ bad of type (ii). Then, in at least one boundary layer $G_i^\delta \cap [E^s]^{[\varepsilon_1 \delta]}$ inequality (1) holds. In this layer, too,

$$\mu(G_i^\delta \cap \mathcal{R}^{[\varepsilon_2 \delta]}) \leq \varepsilon_2 \delta^2$$

by simple geometry. By (1)

$$\mu((G_i^\delta \cap [E^s]^{[\varepsilon_1 \delta]}) \setminus U_{ic}) > \frac{3\varepsilon_1}{4} \mu(G_i^\delta)$$

Because of (2) then

$$\mu(G_i^\delta \cap U_\omega^b) \geq \mu(G_i^\delta \cap [E^s]^{[\varepsilon_1 \delta]} \cap U_\omega^b)) \geq \frac{3\varepsilon_1}{4} \delta^2 - \varepsilon_2 \delta^2 \tag{3}$$

You prescribe ε_1 arbitrarily. I select $\varepsilon_2 < \frac{\varepsilon_1}{2}$. Then by (3) for a bad parallelogram of type (ii)

$$\mu(G_i^\delta \cap U_\omega^b) \geq \frac{\varepsilon_1}{2}\mu(G_i^\delta)$$

All in all

$$4\mu(U_\omega^b) \geq \Sigma^{'}\mu(G_i^\delta \cap U_\omega^b) \geq \frac{\varepsilon_1}{2}\Sigma^{'}\mu(G_i^\delta)$$

where the summation $\Sigma^{'}$ is taken for bad parallelograms of type (ii) (the constant 4 appears because of the regularity of the covering). Finally by the tail bound we obtain the required inequality for the measure of the union of bad parallelograms of type (ii).