1. Show that if $A > 0$, then the inverse of the Lyapunov operator $L_A(X) = A^*X + XA$ is a positive map.

2. Prove that if $\|\phi(I)\| = \|\phi\|$ and $\phi(I) \geq 0$ holds for a linear functional $\phi : M_n \to \mathbb{C}$, then it is positive.

3. Let ϕ be a linear map. Show that ϕ is positive iff ϕ^* is positive, and ϕ is unital iff ϕ^* is trace preserving

4. Let ϕ be a completely positive unital map. Show that if f is convex, then

$$\mathrm{Tr}\, f(\phi(A)) \leq \mathrm{Tr}\, \phi(f(A)).$$

   Moreover, if f is matrix convex, then the inequality holds without the trace as well.

Deadline: Dec 8.