1. Show that in the context of the minimax principle

$$\lambda_k = \max\left\{\min\left\{\langle v, Ay\rangle | v \in \mathcal{K}, \|v\| = 1\right\} k \leq \mathcal{H}, \dim \mathcal{K} = k\right\}$$

holds.

2. Prove the Cauchy interlacing theorem, that is if $A = \begin{pmatrix} B & C \\ D & E \end{pmatrix}$ is an $n \times n$ complex matrix, where B is an $(n-k) \times (n-k)$ matrix, then

$$\lambda_j^{\downarrow}(A) \geq \lambda_j^{\downarrow}(B) \geq \lambda_{j+k}^{\downarrow}$$

for all $j = 1, 2, \ldots, n-k$.

3. Given $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, calculate M^{-1} in terms of the Schur complement $S = A - BD^{-1}C$. Using the Schur factorisation, find the condition of $M \geq 0$.

4. Let P and Q be ortho-projections. Show, that $P \leq Q$ if and only if $PQ = P$.

Deadline: October 6.