Let D and E be diagonal, A and B arbitrary square matrices, x and y complex vectors. Let $A \circ B$ denote the Hadamard product of A and B. Prove the following:

1.
$$\begin{aligned} D(A \circ B)E &= (DAE) \circ B \\ &= (DA) \circ (BE) \\ &= (AE) \circ (DB) \\ &= A \circ (DBE) \end{aligned}$$

2.
$$\left[A\mathrm{Diag}(x)B^T\right]_{ii} = [(a \circ B)x]_i$$

3.
$$\langle y, (A \circ B)x \rangle = \mathrm{Tr}(\mathrm{Diag}(y)^* A \, \mathrm{Diag}(x) B^T)$$

4. Using the previous calculations, show that the Hilbert-Schmidt norm is submultiplicative with the Hadamard product, that is
$$\|A \circ B\| \leq \|A\| \|B\|$$

Deadline: October 13.