1. Prove, that for all $A \in M_n, B \in M_m$ and function f, there is a polynomial $p_{A,B}$, such that $f(A) = p_{A,B}(A)$ and $f(B) = p_{A,B}(B)$.
2. Show, that for all f, $A \in \mathbb{C}^{m \times n}$, $B \in \mathbb{C}^{n \times m}$

$$Af(BA) = f(AB)A$$

   holds.
3. With A and B as in (2), show that if $\exists (BA)^{-1}$, then

$$f(\alpha I_m + AB) = f(\alpha)IB_m + A(BA)^{-1}(f(\alpha I_n + BA) - f(\alpha)I_n)B$$

4. $(I + AB)^{-1} = I - A(I + BA)^{-1}$
5. for $u, v, x, y \in \mathbb{C}^n$, what is $f(\alpha I_n + |u\rangle\langle v| + |x\rangle\langle y|)$?

Deadline: November 3.