1. Let $\beta > 0$, and $H = H^*$ fixed. The free energy

$$F(D) = \operatorname{Tr} DH - \frac{1}{\beta} S(D)$$

is defined on density matrices. Find the density that minimizes F!

2. Prove the Hadamard inequality, that is for any matrix N with columns $v_i \neq 0$ the inequality

$$|\operatorname{Det} N| \leq \prod_i \|v_i\|$$

holds, and there is equality if and only if $\langle v_i, v_j \rangle = 0 \ \forall i \neq j$.

3. Let $M = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}$, and $f : \mathbb{R}^+ \to \mathbb{R}$ a convex function. Show that

$$\operatorname{Tr} f(M) \geq \operatorname{Tr} f(A) + \operatorname{Tr} f(C).$$

Deadline: November 24.