

1. Show with counterexamples, that $f(t) = r^t$ is not matrix monotone for any $r \geq 2$.
2. Show with a counterexample, that $f(t) = t^3$ is not matrix convex
3. Use induction and matrix monotonicity of the square-root to prove the Löwner-Heinz inequality
4. Let $\epsilon > 0$. Show that there is a $\lambda > 0$ real number, such that

$$\begin{pmatrix} A & C \\ C^* & B \end{pmatrix} \leq \begin{pmatrix} A + \epsilon I & 0 \\ 0 & \lambda I \end{pmatrix}$$

Deadline: Dec 1.