1. Show that the geometric mean G is monotone for three variables as well, that is for matrices $A_1 \geq A_2, B_1 \geq B_2, C_1 \geq C_2$ the inequality $G(A_1, B_1, C_1) \geq G(A_2, B_2, C_2)$ holds.

2. Show that for the arithmetic mean A and the harmonic mean H the inequalitiy $H(A, B, C) \leq G(A, B, C) \leq A(A, B, C)$ holds.

3. Let L be a connection. Prove, that $L(M(A, B), M(C, D)) \leq M(L(A, C), L(B, D))$ for all connections M and matrices A, B, C, D if and only if $L = a\omega_l + b\omega_r$ with $a, b \geq 0$. (Hint: use the arithmetic mean to prove the midpoint convexity of f_L)

Deadline: Dec 1.