G2 Exam Questions

2022

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1 Linear Algebra

1.1 Operations on Matrices

1. Can we calculate the sum of a 2×2 and a 3×3 matrix?
2. Can we calculate the product of a 2×2 and a 3×3 matrix?
3. Can we calculate the sum of a 2×3 and a 3×2 matrix?
4. Can we calculate the product of a 2×3 and a 3×2 matrix?
5. For any $n \times n$ matrices A and B, $A + B = B + A$.
6. For any $n \times n$ matrices A and B, $A \cdot B = B \cdot A$.
7. For any $n \times n$ matrix A, we have $A + I = A$ where I is the identity matrix of size $n \times n$.
8. For any $n \times n$ matrix A, we have $A \cdot I = A$ where I is the identity matrix of size $n \times n$.
9. The transpose of an $n \times n$ matrix is an $n \times n$ matrix.
10. For any $n \times n$ matrices A and B we have $(A \cdot B)^T = A^T \cdot B^T$.
11. For any $n \times n$ matrices A and B we have $(A \cdot B)^T = B^T \cdot A^T$.
12. For any $n \times n$ matrices A and B we have $(A + B)^T = A^T + B^T$.
13. For any $n \times n$ matrices A and B we have $(A + B)^T = B^T + A^T$.
14. The product of diagonal matrices is a diagonal matrix.
15. The transpose of a diagonal matrix is a diagonal matrix.
16. The product of upper-triangular matrices is an upper-triangular matrix.
17. The sum of upper-triangular matrices is an upper-triangular matrix.
18. The product of lower-triangular matrices is a lower-triangular matrix.
19. The sum of lower-triangular matrices is a lower-triangular matrix.
20. The transpose of an upper triangular matrix is an upper triangular matrix.
21. The transpose of a lower triangular matrix is an upper triangular matrix.
22. The transpose of a lower triangular matrix is an upper triangular matrix.
23. The transpose of a lower triangular matrix is a lower triangular matrix.

1.2 Determinants

24. The determinant of a real-valued square matrix is always a real number.
25. The determinant of a real-valued square matrix can be a non-real (complex) number.
26. The minor corresponding to a given element $a_{i,j}$ is the determinant of such a matrix which is constructed from the original one by deleting the row and column of the corresponding element $a_{i,j}$.
27. For any $n \times n$ matrices A and B, we have $\det(A + B) = \det(A) + \det(B)$.
28. For any $n \times n$ matrices A and B, we have $\det(A \cdot B) = \det(A) + \det(B)$.
29. For any $n \times n$ matrices A and B, we have $\det(A + B) = \det(A) \cdot \det(B)$.
30. For any $n \times n$ matrices A and B, we have $\det(A \cdot B) = \det(A) \cdot \det(B)$.
31. The determinant of a matrix is not changed if we add one of the rows to another one multiplied by some number.
32. The determinant of a matrix is changed if we add one of the rows to another one multiplied by some number c, and the determinant is multiplied by this number c.
33. The determinant of a matrix is changed if we multiply one of the rows of the matrix by some (non-zero) number c, and the determinant is multiplied by this number c.
34. The determinant of a matrix is not changed if we multiply one of the rows of the matrix by some (non-zero) number c.
35. The determinant of a matrix is not changed if we change the order of two rows.
36. The determinant of a matrix is multiplied by (-1) if we change the order of two rows.
37. The determinant of a diagonal matrix is the product of its elements in its main diagonal.
38. The determinant of an upper-triangular matrix is the product of its elements in its main diagonal.
39. The determinant of an upper-triangular matrix is the product of its elements in its first row.
40. The determinant of an upper-triangular matrix is the product of its elements in its last column.
41. The determinant of a lower-triangular matrix is the product of its elements in its main diagonal.
42. The determinant of a lower-triangular matrix is the product of its elements in its first column.
43. The determinant of a lower-triangular matrix is the product of its elements in its last row.

1.3 Rank of a matrix

44. Vectors $v_1, v_2 \dots v_n$ are linearly independent if the equation
$$c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0$$
can only hold if $c_1 = c_2 = \cdots = c_n = 0$.
45. Vectors $v_1, v_2 \dots v_n$ are linearly dependent if the equation
$$c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0$$
can only hold if $c_1 = c_2 = \cdots = c_n = 0$.
46. If two vectors are linearly dependent, then they are on the same line.
47. If two vectors are linearly independent, then they are on the same line.

48. If two vectors are linearly dependent, then they cannot be on the same line.
49. If two vectors are linearly independent, then they cannot be on the same line.
50. If three vectors are linearly dependent, then they are on the same line.
51. If three vectors are linearly dependent, then they are on the same plane.
52. If three vectors are linearly independent, then they are on the same plane.
53. The column rank of a matrix is the number of its linearly independent columns.
54. The column rank of a matrix is the number of its linearly dependent columns.
55. The row rank of a matrix is the number of its linearly dependent rows.
56. The row rank of a matrix is the number of its linearly independent rows.
57. For an $n \times n$ matrix, the row rank is always the same as the column rank.
58. For an $n \times n$ matrix, the row rank is always smaller than the column rank.
59. For an $n \times n$ matrix, the row rank is always bigger than the column rank.
60. For an $n \times m$ matrix where $n < m$, the row rank is always smaller than the column rank.
61. For an $n \times m$ matrix where $n < m$, the row rank is always bigger than the column rank.

1.4 Inverse of a matrix

62. If A is an $n \times n$ matrix, then there is always such a matrix B for which $A \cdot B = A$.
63. If A is an $n \times n$ matrix, then there is always such a matrix B for which $B \cdot A = A$.
64. If A is an $n \times n$ matrix, then there is always such a matrix B for which $A \cdot B = I$ (where I is the $n \times n$ identity matrix).
65. If A is an $n \times n$ matrix, then there is always such a matrix B for which $B \cdot A = I$ (where I is the $n \times n$ identity matrix).
66. If A is an $n \times n$ matrix, then there is such a matrix B for which $B \cdot A = I$ (where I is the $n \times n$ identity matrix) if and only if $\det(A) = 0$.
67. If A is an $n \times n$ matrix, then there is such a matrix B for which $B \cdot A = I$ (where I is the $n \times n$ identity matrix) if and only if $\det(A) \neq 0$.
68. If A is an $n \times n$ matrix, then there is such a matrix B for which $B \cdot A = I$ (where I is the $n \times n$ identity matrix) if and only if $\det(A) = 1$.
69. If A is an $n \times n$ matrix, then there is such a matrix B for which $B \cdot A = I$ (where I is the $n \times n$ identity matrix) if and only if $\det(A) \neq 1$.
70. For $n \times n$ matrices A and B for which $\det(A) \neq 0$ and $\det(B) \neq 0$ we have $(A+B)^{-1} = A^{-1} + B^{-1}$.
71. For $n \times n$ matrices A and B for which $\det(A) \neq 0$ and $\det(B) \neq 0$ we have $(A+B)^{-1} = B^{-1} + A^{-1}$.
72. For $n \times n$ matrices A and B for which $\det(A) \neq 0$ and $\det(B) \neq 0$ we have $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$.
73. For $n \times n$ matrices A and B for which $\det(A) \neq 0$ and $\det(B) \neq 0$ we have $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.

1.5 Systems of Linear Algebraic Equations (SLAE)

74. When we are doing the Gaussian elimination, the right-hand side should always be changed along with the left-hand side.
75. When we are doing the Gaussian elimination, the right-hand side should never be changed along with the left-hand side.
76. The extended matrix is in an echelon form when on the left-hand side we have an upper triangular matrix.
77. The rank of a matrix is not changed if we add one of the rows to another one multiplied by some number.
78. The rank of a matrix is changed if we add one of the rows to another one multiplied by some number c, and the rank is multiplied by this number c.
79. The rank of a matrix is changed if we add one of the rows to another one multiplied by some number c, and the rank is increased by one.
80. The rank of a matrix is changed if we add one of the rows to another one multiplied by some number c, and the rank is decreased by one.
81. The rank of a matrix is changed if we multiply one of the rows of the matrix by some (non-zero) number c, and the rank is multiplied by this number c.
82. The rank of a matrix is not changed if we multiply one of the rows of the matrix by some (non-zero) number c.
83. The rank of a matrix is not changed if we change the order of two rows.
84. The rank of a matrix is decreased by 1 if we change the order of two rows.
85. The rank of a matrix is increased by 1 if we change the order of two rows.
86. The number of solutions is not changed if we add one of the rows to another one multiplied by some number.
87. The number of solutions can change if we add one of the rows to another one multiplied by some number.
88. The number of solutions is not changed if we multiply one of the rows by some (non-zero) number.
89. The number of solutions is not changed if we multiply one of the rows by some (non-zero) number.
90. The inverse of an $n \times n$ matrix A^{-1} can be calculated by using the Gaussian elimination in a way that we start from the extended matrix $(A|I)$ in the end the extended matrix should have the form $(I|B)$ and here $A^{-1} = B$ (where I is the $n \times n$ identity matrix).
91. The inverse of an $n \times n$ matrix A^{-1} can be calculated by using the Gaussian elimination in a way that we start from the extended matrix $(A|I)$ in the end the extended matrix should have the form $(I|B)$ and here $A^{-1} = \dfrac{1}{\det(A)}B$ (where I is the $n \times n$ identity matrix).

    item The inverse of an $n \times n$ matrix A^{-1} can be calculated by using the Gaussian elimination in a way that we start from the extended matrix $(A|I)$ in the end the extended matrix should have the form $(I|B)$ and here $A^{-1} = \dfrac{1}{\det(B)}B$ (where I is the $n \times n$ identity matrix).
92. The inverse of an $n \times n$ matrix A^{-1} can be calculated by using the Gaussian elimination in a way that we start from the extended matrix $(A|I)$ in the end the extended matrix should have the form $(0|B)$ and here $A^{-1} = B$ (where 0 is the $n \times n$ all-zero matrix).
93. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if $\det(A) = 0$.

94. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if $\det(A) \neq 0$.
95. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if $\det(A) > 0$.
96. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if $\det(A) < 0$.
97. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if the rank of A is n.
98. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if the inverse of A exists.
99. Let A be an $n \times n$ matrix. Then, the SLAE $Ax = b$ has a unique solution if and only if the transpose of A exists.
100. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A})$, the equation has a unique solution.
101. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A}) = n$ (where n is the number of variables), the equation has a unique solution.
102. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A}) < n$ (where n is the number of variables), the equation has a unique solution.
103. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A}) < n$ (where n is the number of variables), the equation has no solution.
104. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A}) < n$ (where n is the number of variables), the equation has infinitely many solutions.
105. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) = \text{rank}(\widehat{A}) > n$ (where n is the number of variables), the equation has a unique solution.
106. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) < \text{rank}(\widehat{A})$ (where n is the number of variables), the equation has no solution.
107. Let us consider the SLAE $Ax = b$ and let $\widehat{A}$ be the extended matrix $(A|b)$. Then, if $\text{rank}(A) < \text{rank}(\widehat{A})$ (where n is the number of variables), the equation has infinitely many solutions.
108. A homogeneous system of linear equations is such a system of linear equation $Ax = b$ for which $b = \mathbf{0}$ (the all-zero vector).
109. A homogeneous system of linear equations is such a system of linear equation $Ax = b$ for which $b = \mathbf{1}$ (the all-one vector).
110. A homogeneous system of linear equations has no solutions.
111. A homogeneous system of linear equations has always at least one solution.
112. A homogeneous system of linear equations has always a unique solution, which is the all-zero vector.
113. A homogeneous system of linear equations has always a unique solution, which is the all-one vector.
114. A homogeneous system of linear equations has always infinitely many solutions.
115. A homogeneous system of linear equations $Ax = \mathbf{0}$ has infinitely many solutions if $\det(A) = 0$.
116. A homogeneous system of linear equations $Ax = \mathbf{0}$ has infinitely many solutions if $\det(A) \neq 0$.
117. A homogeneous system of linear equations $Ax = \mathbf{0}$ has no solutions if $\det(A) = 0$.

1.6 Eigenvalues and eigenvectors

118. An $n \times n$ matrix A with real elements always has n-many different real eigenvalues.
119. An $n \times n$ matrix A with real elements will always have n-many different (real or complex) eigenvalues.
120. An $n \times n$ matrix A with real elements can have complex eigenvalues.
121. An eigenvalue with multiplicity of 1 has always one corresponding eigenvector.
122. An eigenvalue with multiplicity of 1 has infinitely many corresponding eigenvectors, and they are linearly dependent.
123. An eigenvalue with multiplicity of 1 has infinitely many corresponding eigenvectors, and they are linearly independent.
124. An eigenvalue with multiplicity of k has infinitely many corresponding eigenvectors, and there are always k-many linearly independent ones among them.
125. An eigenvalue with multiplicity of k has infinitely many corresponding eigenvectors, and there can be either $1, 2, 3, \ldots$ or k-many linearly independent ones among them.
126. The determinant of a matrix is the product of its eigenvalues (calculated with multiplicity).
127. The determinant of a matrix is the sum of its eigenvalues (calculated with multiplicity).
128. If the determinant of a matrix is zero, then zero is its eigenvalue.
129. If the determinant of a matrix is zero, then the sum of all of its eigenvalues is zero.
130. For a diagonal matrix, its eigenvalues are in its main diagonal.
131. For an identity matrix, its one eigenvalue is 1.
132. For an $n \times n$ identity matrix, its eigenvalues are 1 and $n-1$-many zeros.
133. If A is a symmetric matrix, all of its eigenvalues are real.
134. If A is a symmetric matrix, all of its eigenvalues are in the form $\lambda_1, \frac{1}{\lambda_1}, \lambda_2, \frac{1}{\lambda_2}, \ldots, \lambda_n, \frac{1}{\lambda_n}$.
135. If A is a symmetric matrix, all of its eigenvalues are in the form $\lambda_1, -\lambda_1, \lambda_2, -\lambda_2, \ldots, \lambda_n, -\lambda_n$.
136. If A and B are similar matrices, then all of its eigenvalues (and the corresponding multiplicities) are the same.
137. A is called a diagonalizable matrix if it is similar to a diagonal matrix.
138. A is called a diagonalizable matrix if its inverse is a diagonal matrix.
139. If A is symmetric and real, then it is diagonalizable.
140. If A has n-many different real eigenvalues, then it is diagonalizable.
141. If A is diagonalizable and symmetric, then it can be written in the form $A = V \cdot D \cdot V^{-1}$ where D is a diagonal matrix with the eigenvalues in the main diagonal and V has the eigenvectors as its columns.
142. If A is diagonalizable and symmetric, then it can be written in the form $A = D \cdot V \cdot D^{-1}$ where D is a diagonal matrix with the eigenvalues in the main diagonal and V has the eigenvectors as its columns.

1.7 Linear spaces (Vector spaces)

143. The set of complex numbers forms a vector space.

144. The set of continuous functions forms a vector space.

145. The set of the solutions of a given linear system of algebraic equations forms a vector space.

146. The set of the solutions of a given homogeneous linear system of algebraic equations forms a vector space.

147. For vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\\0\end{pmatrix}$, the generated subspace has vectors in the form $\begin{pmatrix}a\\b\\0\end{pmatrix}$ $(a, b \in \mathbb{R})$.

148. For vectors $v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ and $v_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$, the vector $\begin{pmatrix}0\\a\\b\end{pmatrix}$ $(a, b \in \mathbb{R})$ is in the Span of v_1 and v_2.

149. The basis of a vector space is a set of linearly dependent vectors that generate the given set.

150. The basis of a vector space is a set of linearly independent vectors that generate the given set.

151. The orthogonal basis of a vector space is a set of vectors which are orthogonal to the all-one vector.

152. The orthogonal basis of a vector space is a set of vectors which are orthogonal to the all-zero vector.

153. The orthogonal basis of a vector space is a set of vectors which are orthogonal to each other.

154. The orthonormal basis of a vector space is an orthogonal basis of vectors which all have a length of one.

155. The orthonormal basis of a vector space is an orthogonal basis of vectors which all have a length of zero.

1.8 Linear operators

156. If $L : V \to W$ is a linear operator (and V and W are vector spaces), then $L(v_1 + v_2) = L(v_1) + L(v_2)$ for any vectors $v_1, v_2 \in V$.

157. If $L : V \to W$ is a linear operator (and V and W are vector spaces), then $L(v_1 \cdot v_2) = L(v_1) + L(v_2)$ for any vectors $v_1, v_2 \in V$.

158. If $L : V \to W$ is a linear operator (and V and W are vector spaces), then $L(v_1 \cdot v_2) = L(v_1) \cdot L(v_2)$ for any vectors $v_1, v_2 \in V$.

159. If $L : V \to W$ is a linear operator (and V and W are vector spaces), then $L(v_1 + v_2) = L(v_1) \cdot L(v_2)$ for any vectors $v_1, v_2 \in V$.

160. If $L : V \to W$ is a linear operator (and V and W are real vector spaces), then $L(c \cdot v) = c \cdot L(v)$ for any vector $v \in V$ and any constant $c \in \mathbb{R}$.

161. If $L : V \to W$ is a linear operator (and V and W are real vector spaces), then $L(c + v) = c \cdot L(v)$ for any vector $v \in V$ and any constant $c \in \mathbb{R}$.

162. If $L : V \to W$ is a linear operator (and V and W are real vector spaces), then $L(c + v) = c + L(v)$ for any vector $v \in V$ and any constant $c \in \mathbb{R}$.

163. If $L : V \to W$ is a linear operator (and V and W are real vector spaces), then $L(c \cdot v) = c + L(v)$ for any vector $v \in V$ and any constant $c \in \mathbb{R}$.

164. The matrix of a linear operator remains the same even if we change the basis of the vector space.

165. The matrix of a linear operator might change if we change the basis of the vector space.

166. The limit is a linear operator.

167. The limit is not a linear operator.

168. Differentiation is not a linear operator.

169. Differentiation is a linear operator.

170. Integration is not a linear operator.

171. Integration is a linear operator.

172. The kernel of a transformation is the set of those vectors whose image is the zero vector.

173. The kernel of a transformation is the set of those vectors which are the images of the zero vector.

174. The kernel of a linear transformation is always the origin only.

175. The rank nullity theorem states that for any linear operator $L : V \to V$ we have $\dim(\mathrm{Ker}(L)) + \dim(\mathrm{Im}(L)) = \dim(V)$.

176. The rank nullity theorem states that for any linear operator $L : V \to V$ we have $\dim(\mathrm{Ker}(L)) \cdot \dim(\mathrm{Im}(L)) = \dim(V)$.

177. The rank nullity theorem states that for any linear operator $L : V \to V$ we have $\dim(\mathrm{Ker}(L)) + \dim(\mathrm{Im}(L)) > \dim(V)$.

178. The rank nullity theorem states that for any linear operator $L : V \to V$ we have $\dim(\mathrm{Ker}(L)) + \dim(\mathrm{Im}(L)) < \dim(V)$.

2 Numerical Series

179. A numerical series converges if the sequence of the partial sums converges.

180. The sum of a numerical series is always the same even if we change the order of the elements.

181. The sum of a convergent numerical series is always the same even if we change the order of the elements.

182. If $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 0$.

183. If $\displaystyle\sum_{n=0}^{\infty} a_n$ converges, then $\displaystyle\lim_{n\to\infty} a_n = 1$.

184. If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

185. If $\displaystyle\lim_{n\to\infty} a_n < 1$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ converges.

186. If $\displaystyle\lim_{n\to\infty} a_n \neq 0$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges.

187. If $\displaystyle\lim_{n\to\infty} a_n = 1$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges.

188. If $\displaystyle\lim_{n\to\infty} a_n > 1$, then $\displaystyle\sum_{n=0}^{\infty} a_n$ diverges.

189. The sequence $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ converges, if $\alpha > 1$.

190. The sequence $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ converges, if $\alpha < 1$.

191. The sequence $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ diverges, if $\alpha > 1$.

192. The sequence $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ can be either convergent or divergent if $\alpha = 1$.

193. The series $\sum_{n=1}^{\infty} q^n$ converges, if $|q| < 1$.

194. The series $\sum_{n=1}^{\infty} q^n$ converges, if $-1 < q < 1$.

195. The series $\sum_{n=1}^{\infty} q^n$ converges, if $q > 1$.

196. The series $\sum_{n=1}^{\infty} q^n$ converges, if $q < 1$.

197. If the series $\sum_{n=1}^{\infty} q^n$ converges, then $\sum_{n=1}^{\infty} q^n = \frac{1}{1-q}$.

198. If the series $\sum_{n=1}^{\infty} q^n$ converges, then $\sum_{n=1}^{\infty} q^n = \frac{1}{1+q}$.

199. If the series $\sum_{n=1}^{\infty} q^n$ converges, then $\sum_{n=1}^{\infty} q^n = \frac{1}{q-1}$.

200. If for an alternating series $\sum_{n=0}^{\infty} (-1)^n a_n$ $(a_n \geq 0)$ the sequence a_n is monotonically decreasing and it tends to zero, then the series is convergent.

201. If for an alternating series $\sum_{n=0}^{\infty} (-1)^n a_n$ $(a_n \geq 0)$ the sequence a_n tends to zero, then the series is convergent.

202. If for an alternating series $\sum_{n=0}^{\infty} (-1)^n a_n$ $(a_n \geq 0)$ the sequence a_n is monotonically decreasing, then the series is convergent.

203. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$, then the series diverges.

204. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$, then the series converges.

205. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then the series converges.

206. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$, then the series diverges.

207. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \sqrt[n]{a_n} < 1$, then the series diverges.

208. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \sqrt[n]{a_n} < 1$, then the series converges.

209. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \sqrt[n]{a_n} > 1$, then the series diverges.

210. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ the limit $\lim_{n\to\infty} \sqrt[n]{a_n} > 1$, then the series converges.

211. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n > a_n$ and $\sum_{n=0}^{\infty} b_n < \infty$, then $\sum_{n=0}^{\infty} a_n < \infty$.

212. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n > a_n$, then $\sum_{n=0}^{\infty} a_n < \infty$.

213. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n > a_n$ and $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

214. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n < a_n$ and $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

215. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n < a_n$, then $\sum_{n=0}^{\infty} a_n$ diverges.

216. If for a positive series $\sum_{n=0}^{\infty} a_n$ $(a_n > 0)$ there is some other positive series $\sum_{n=0}^{\infty} b_n$ for which $b_n < a_n$ and $\sum_{n=0}^{\infty} b_n < \infty$, then $\sum_{n=0}^{\infty} a_n < \infty$.

217. A series $\sum_{n=0}^{\infty} a_n$ is called absolutely convergent if $\sum_{n=0}^{\infty} |a_n|$ is convergent.

218. A series $\sum_{n=0}^{\infty} a_n$ is called absolutely convergent if $\left|\sum_{n=0}^{\infty} a_n\right|$ is convergent.

219. If a series is absolutely convergent, then it is convergent.

220. If a series is convergent, then it is absolutely convergent.

221. If a series is convergent but not absolutely convergent, then it is called conditionally convergent.

222. If a series is absolutely convergent but not convergent, then it is called conditionally convergent.

223. For the error e_N of the approximation $\sum_{n=0}^{N}(-1)^n a_n \approx \sum_{n=0}^{\infty}(-1)^n a_n$ ($a_n > 0$ for every n) we have $|e_N| \leq |a_{N+1}|$.

224. For the error e_N of the approximation $\sum_{n=0}^{N}(-1)^n a_n \approx \sum_{n=0}^{\infty}(-1)^n a_n$ ($a_n > 0$ for every n) we have $|e_N| \leq |a_N|$.

225. For the error e_N of the approximation $\sum_{n=0}^{N}(-1)^n a_n \approx \sum_{n=0}^{\infty}(-1)^n a_n$ ($a_n > 0$ for every n) we have $|e_N| = |a_{N+1}|$.

226. For the error e_N of the approximation $\sum_{n=0}^{N} a_n \approx \sum_{n=0}^{\infty} a_n$ ($a_n > 0$ for every n) we have $|e_N| \leq |a_{N+1}|$.

3 Function series

227. For a function sequence f_n converging point-wise, $\lim_{n\to\infty} \int_a^b f_n(x) = \int_a^b \lim_{n\to\infty} f_n(x)$ where $f_n(x) : [a,b] \to \mathbb{R}$ continuous functions.

228. For a function sequence f_n converging uniformly, $\lim_{n\to\infty} \int_a^b f_n(x) = \int_a^b \lim_{n\to\infty} f_n(x)$ where $f_n(x) : [a,b] \to \mathbb{R}$ continuous functions.

229. For a function series f_n converging point-wise, $\sum_{n=0}^{\infty} \int_a^b f_n(x) = \int_a^b \sum_{n=0}^{\infty} f_n(x)$ where $f_n(x) : [a,b] \to \mathbb{R}$ continuous functions.

230. For a function series f_n converging uniformly, $\sum_{n=0}^{\infty} \int_a^b f_n(x) = \int_a^b \sum_{n=0}^{\infty} f_n(x)$ where $f_n(x) : [a,b] \to \mathbb{R}$ continuous functions.

3.1 Power series

231. The interval of convergence of a series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ is in the form $(x_0 - R, x_0 + R)$ where $R > 0$.

232. The interval of convergence of a series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ is in the form $[x_0 - R, x_0 + R]$ where $R > 0$.

233. The interval of convergence of a series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ is in the form $(x_0 - a_n, x_0 + a_n)$ where $R > 0$.

234. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ the radius of convergence is $R = 0$, then the series is convergent only at the point x_0.

235. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ the radius of convergence is $R=0$, then the series is divergent everywhere.

236. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ the radius of convergence is $R=\infty$, then the series is convergent everywhere.

237. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ the radius of convergence is $R=\infty$, then the series is convergent only at the point x_0.

238. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ the radius of convergence is $R=\infty$, then the series is divergent everywhere.

239. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $0<\limsup_{n\to\infty}\sqrt{|a_n|}<\infty$, then the radius of convergence is $R=\dfrac{1}{\limsup_{n\to\infty}\sqrt{|a_n|}}$.

240. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $0<\limsup_{n\to\infty}\sqrt{|a_n|}<\infty$, then the radius of convergence is $R=\limsup_{n\to\infty}\sqrt{|a_n|}$.

241. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\limsup_{n\to\infty}\sqrt{|a_n|}=0$, then the radius of convergence is $R=\infty$.

242. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\limsup_{n\to\infty}\sqrt{|a_n|}=\infty$, then the radius of convergence is $R=\infty$.

243. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\limsup_{n\to\infty}\sqrt{|a_n|}=0$, then the radius of convergence is $R=0$.

244. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\limsup_{n\to\infty}\sqrt{|a_n|}=\infty$, then the radius of convergence is $R=0$.

245. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty}\dfrac{|a_{n+1}|}{|a_n|}=\alpha$ where $0<\alpha<\infty$, then the radius of convergence is $R=\dfrac{1}{\alpha}$.

246. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty}\dfrac{|a_{n+1}|}{|a_n|}=\alpha$ where $0<\alpha<\infty$, then the radius of convergence is $R=\alpha$.

247. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty}\dfrac{|a_{n+1}|}{|a_n|}=0$, then the radius of convergence is $R=0$.

248. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty}\dfrac{|a_{n+1}|}{|a_n|}=0$, then the radius of convergence is $R=\infty$.

249. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \infty$, then the radius of convergence is $R = \infty$.

250. If for a power series $\sum_{n=1}^{\infty} a_n(x-x_0)^n$ we have $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \infty$, then the radius of convergence is $R = 0$.

3.2 Taylor series

251. The error of the approximation $\sum_{n=1}^{N} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \approx \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ is given by the formula $\frac{f^{(N+1)}(\xi)}{(N+1)!}(x-x_0)^{N+1}$ where $\xi \in [x_0, x]$.

252. The error of the approximation $\sum_{n=1}^{N} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \approx \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ is given by the formula $\frac{f^{(N)}(\xi)}{(N)!}(x-x_0)^{N}$ where $\xi \in [x_0, x]$.

253. The error of the approximation $\sum_{n=1}^{N} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \approx \sum_{n=1}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$ is given by the formula $\frac{f^{(N+1)}(x_0)}{(N+1)!}(x-x_0)^{N+1}$.

3.3 Fourier series

254. The function $f(x) = \cos(2x) + \sin(x) + 2$ is a trigonometric polynomial.

255. The function $f(x) = \cos(2x) + \sin(x)$ is a trigonometric polynomial.

256. The function $f(x) = x^2 + \cos(x) + \sin(x)$ is a trigonometric polynomial.

257. The sum of two trigonometric polynomials is a trigonometric polynomial.

258. A trigonometric polynomial multiplied by a scalar is also a trigonometric polynomial.

259. Every continuous function periodic by 2π can be written in the form

$$f(x) = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + \dots$$

where $f_1, f_2, f_3, \dots \in \left\{ \frac{1}{\sqrt{2\pi}}, \left(\frac{1}{\sqrt{\pi}}\sin(nx)\right)_{n\in\mathbb{N}}, \left(\frac{1}{\sqrt{\pi}}\cos(nx)\right)_{n\in\mathbb{N}} \right\}$.

260. Every continuous function can be written in the form

$$f(x) = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + \dots$$

where $f_1, f_2, f_3, \dots \in \left\{ \frac{1}{\sqrt{2\pi}}, \left(\frac{1}{\sqrt{\pi}}\sin(nx)\right)_{n\in\mathbb{N}}, \left(\frac{1}{\sqrt{\pi}}\cos(nx)\right)_{n\in\mathbb{N}} \right\}$.

261. If the Fourier series is in the form $a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$ and the function f is odd, then for its Fourier series $a_k = 0$.

262. If the Fourier series is in the form $a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$ and the function f is odd, then for its Fourier series $b_k = 0$.

263. If the Fourier series is in the form $a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$ and the function f is even, then for its Fourier series $a_k = 0$.

264. If the Fourier series is in the form $a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$ and the function f is even, then for its Fourier series $b_k = 0$.

265. If f is continuously differentiable, then its Fourier series $\mathcal{F}f$ at point x_0 can be calculated as
$$(\mathcal{F}f)(x_0) = \frac{\lim_{x\to x_0+0} f(x) + \lim_{x\to x_0-0} f(x)}{2}$$

266. If f is continuously differentiable, then its Fourier series $\mathcal{F}f$ at point x_0 can be calculated as
$$(\mathcal{F}f)(x_0) = \frac{\lim_{x\to x_0+0} f(x) - \lim_{x\to x_0-0} f(x)}{2}$$

267. If f is continuously differentiable, then its Fourier series $\mathcal{F}f$ at point x_0 can be calculated as
$$(\mathcal{F}f)(x_0) = \frac{\lim_{x\to x_0+0} f(x) \cdot \lim_{x\to x_0-0} f(x)}{2}$$

4 Multivariable analysis

4.1 Limits, continuity

268. The set $x^2 + y^2 < 1$ (unit disc without its border) is open.

269. The set $x^2 + y^2 < 1$ (unit disc without its border) is closed.

270. The set $x^2 + y^2 \le 1$ (unit disc with its border) is open.

271. The set $x^2 + y^2 \le 1$ (unit disc with its border) is closed.

272. For any sequence x_k ($k \in \mathbb{N}$) for which $x_k \in \mathbb{R}^2$ ($k \in \mathbb{N}$) and $\lim\limits_{k\to\infty} x_k = x_0$, we have $\lim\limits_{k\to\infty} f(x_k) = f(x_0)$.

273. For any sequence x_k ($k \in \mathbb{N}$) for which $x_k \in \mathbb{R}^2$ ($k \in \mathbb{N}$) and $\lim\limits_{k\to\infty} x_k = x_0$, we have $\lim\limits_{k\to\infty} f(x_k) = f(x_0)$ if and only if f is continuous at x_0.

4.2 Differentiation

274. The total derivative of a function $f : \mathbb{R}^n \to \mathbb{R}$ is a vector containing all the partial derivatives of f.

275. The total derivative of a function $f : \mathbb{R}^n \to \mathbb{R}$ is the sum of all the partial derivatives of f.

276. The gradient of a function $f : \mathbb{R}^n \to \mathbb{R}$ is a vector containing all the partial derivatives of f.

277. A level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ are the curves where $f(x) = c$ for a given value of $c \in \mathbb{R}$.

278. A level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ are the curves where $f(c) = x$ for a given value of $c \in \mathbb{R}$.

279. If a function $f : \mathbb{R}^n \to \mathbb{R}$ is totally differentiable at point $a \in \mathbb{R}^n$, then it is continuous at a.

280. If a function $f : \mathbb{R}^n \to \mathbb{R}$ is continuous at point $a \in \mathbb{R}^n$, then it is totally differentiable at a.

281. A function $f : \mathbb{R}^n \to \mathbb{R}$ is totally differentiable at point a if and only if f is continuous at point a.

282. If for a function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ all of its second partial derivatives exist and they are continuous, then $\dfrac{\partial^2 f(x,y)}{\partial x \partial y} = \dfrac{\partial^2 f(x,y)}{\partial y \partial x}$.

283. If for a function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ all of its second partial derivatives exist and they are continuous, then $\dfrac{\partial^2 f(x,y)}{\partial x \partial x} = \dfrac{\partial^2 f(x,y)}{\partial y \partial y}$.

284. There exists such a function $f(x,y):\mathbb{R}^2\to\mathbb{R}$ for which all of its second partial derivatives exist and they are continuous, and $\dfrac{\partial f(x,y)}{\partial^2 x\partial y}\neq\dfrac{\partial^2 f(x,y)}{\partial y\partial x}$.

285. The directional derivative of a function $f:\mathbb{R}^n\to\mathbb{R}$ at a given point a and in a given direction v can be given by $\partial_v f=\mathrm{grad}(f)\cdot\dfrac{v}{|v|}$ where $\cdot$ is a scalar product and $|v|$ is the length of vector v.

286. The directional derivative of a function $f:\mathbb{R}^n\to\mathbb{R}$ at a given point a and in a given direction v can be given by $\partial_v f=\mathrm{grad}(f)\cdot\dfrac{v}{|v|}$ where $\cdot$ is a vector product and $|v|$ is the length of vector v.

287. The directional derivative of a function $f:\mathbb{R}^n\to\mathbb{R}$ at a given point a and in a given direction v can be given by $\partial_v f=\mathrm{grad}(f)\cdot\dfrac{|v|}{v}$ where $\cdot$ is a scalar product and $|v|$ is the length of vector v.

288. The directional derivative of a function $f:\mathbb{R}^n\to\mathbb{R}$ at a given point a and in a given direction v can be given by $\partial_v f=|\mathrm{grad}(f)|\cdot\dfrac{|v|}{v}$ where $\cdot$ is a scalar product and $|.|$ denotes the length of a vector.

4.3 Extrema

289. The Hessian (or Jacobian) of a function $f:\mathbb{R}^n\to\mathbb{R}$ is given by the matrix

$$\begin{pmatrix}\dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_1\partial x_n}\\ \dfrac{\partial^2 f}{\partial x_2\partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} & \cdots & \dfrac{\partial^2 f}{\partial x_2\partial x_n}\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial^2 f}{\partial x_n\partial x_1} & \dfrac{\partial^2 f}{\partial x_n\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_n^2}\end{pmatrix}$$

290. The Hessian (or Jacobian) of a function $f:\mathbb{R}^n\to\mathbb{R}$ is given by the matrix

$$\begin{pmatrix}\dfrac{\partial f}{\partial x_1} & \dfrac{\partial f}{\partial x_2} & \cdots & \dfrac{\partial f}{\partial x_n}\\ \dfrac{\partial f}{\partial x_2} & \dfrac{\partial f}{\partial x_3} & \cdots & \dfrac{\partial f}{\partial x_{n-1}}\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial f}{\partial x_n} & \dfrac{\partial f}{\partial x_{n-1}} & \cdots & \dfrac{\partial f}{\partial x_1}\end{pmatrix}$$

291. The Hessian (or Jacobian) of a function $f:\mathbb{R}^n\to\mathbb{R}$ is given by the matrix

$$\begin{pmatrix}\dfrac{\partial f}{\partial x_1} & \dfrac{\partial f}{\partial x_2} & \cdots & \dfrac{\partial f}{\partial x_n}\\ \dfrac{\partial f}{\partial x_1} & \dfrac{\partial f}{\partial x_2} & \cdots & \dfrac{\partial f}{\partial x_n}\\ \vdots & \vdots & \ddots & \vdots\\ \dfrac{\partial f}{\partial x_1} & \dfrac{\partial f}{\partial x_2} & \cdots & \dfrac{\partial f}{\partial x_n}\end{pmatrix}$$

292. The Hessian (or Jacobian) matrix of a function $f:\mathbb{R}^n\to\mathbb{R}$ is always symmetric.

293. If p is a local extermum of a function $f:\mathbb{R}^n\to\mathbb{R}$, then p is a critical point of f (meaning that all of its partial derivatives are zero).

294. If p is a a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), then p is a local extermum of f.

295. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is positive and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) > 0$, then p is a local minimum.

296. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is positive and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) > 0$, then p is a local maximum.

297. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is positive and $\dfrac{\partial^2 f(x,y)}{\partial x^2} < 0$, then p is a local minimum.

298. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is positive and $\dfrac{\partial^2 f(x,y)}{\partial x^2} < 0$, then p is a local maximum.

299. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) < 0$, then p is a local minimum.

300. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) < 0$, then p is a local maximum.

301. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) > 0$, then p is a local maximum.

302. If p is a critical point of function $f : \mathbb{R}^2 \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative and $\dfrac{\partial^2 f(x,y)}{\partial x^2}(p) < 0$, then p is a local minimum.

303. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative, then p is not a local extremum.

304. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is negative, then p can be either a local maximum, local minimum or neither.

305. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is positive, then p is not a local extremum.

306. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is zero, then p is not a local extremum.

307. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the determinant of the Hessian is zero, then p can be either a local maximum, local minimum or neither.

308. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and all the eigenvalues of the Hessian are positive, then p is a local maximum.

309. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and all the eigenvalues of the Hessian are positive, then p is a local minimum.

310. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and all the eigenvalues of the Hessian are negative, then p is a local maximum.

311. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and all the eigenvalues of the Hessian are negative, then p is a local minimum.

312. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has both positive and negative eigenvalues, then p is a local minimum.

313. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has both positive and negative eigenvalues, then p is a local maximum.

314. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has both positive and negative eigenvalues, then p is not a local extremum.

315. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has both positive and negative eigenvalues, then p can be either a local maximum, minimum or neither.

316. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has at least one zero eigenvalue, then p is not a local extremum.

317. If p is a critical point of function $f : \mathbb{R}^n \to \mathbb{R}$ (meaning that all of its partial derivatives are zero), and the Hessian has at least one zero eigenvalue, then p can be either a local maximum, minimum or neither.

318. If $A \subset \mathbb{R}^n$ is a closed and bounded set, then $f : A \to \mathbb{R}$ is a bounded function and there are such points $\underline{a}, \underline{b} \in A$ for which $\max\limits_A f(\underline{x}) = f(\underline{a})$ and $\min\limits_A f(\underline{x}) = f(\underline{b})$.

319. If $A \subset \mathbb{R}^n$ is a closed and bounded set, then if $f : A \to \mathbb{R}$ is continuous, then it is also bounded, and there are such points $\underline{a}, \underline{b} \in A$ for which $\max\limits_A f(\underline{x}) = f(\underline{a})$ and $\min\limits_A f(\underline{x}) = f(\underline{b})$.

320. If p is a local conditional extrema of $f : \mathbb{R}^n \to \mathbb{R}$ on the boundary of set A, then all the partial derivatives of funcion f should be zero at p.

321. If p is a local conditional extrema of $f : \mathbb{R}^n \to \mathbb{R}$ on the boundary of set A, then all the partial derivatives of funcion $L : \mathbb{R}^n \to \mathbb{R}$ should be zero at p where $L(\underline{x}) = f(\underline{x}) - \lambda(g(\underline{x}) - b)$ and the set A is described by the equation $g(\underline{x}) \leq b$.

322. If p is a local conditional extrema of $f : \mathbb{R}^n \to \mathbb{R}$ on the boundary of set A, then all the partial derivatives of funcion $L : \mathbb{R}^n \to \mathbb{R}$ should be zero at p where $L(\underline{x}) = (g(\underline{x}) - b) - \lambda f(\underline{x})$ and the set A is described by the equation $g(\underline{x}) \leq b$.

323. The conditional extrema of a function $f : \mathbb{R}^n \to \mathbb{R}$ on a bounded and closed set A are either points on the boundary of the set or local extrema inside the set A.

324. The conditional extrema of a function $f : \mathbb{R}^n \to \mathbb{R}$ on a bounded and closed set A are local extrema inside the set A.

325. The conditional extrema of a function $f : \mathbb{R}^n \to \mathbb{R}$ on a bounded and closed set A are always on the boundary of set A.

4.4 Integration

326. An upper Riemann sum of a function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}^2$, bounded, measurable) on set A is always bigger (or equal) than the double integral $\iint\limits_A f(x, y)dxdy$.

327. An upper Riemann sum of a function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}^2$, bounded, measurable) on set A is always smaller (or equal) than the double integral $\iint\limits_A f(x, y)dxdy$.

328. A lower Riemann sum of a function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}^2$, bounded, measurable) on set A is always smaller (or equal) than the double integral $\iint\limits_A f(x, y)dxdy$.

329. A lower Riemann sum of a function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}^2$, bounded, measurable) on set A is always bigger (or equal) than the double integral $\iint\limits_A f(x,y)dxdy$.

330. If $\iint\limits_{B_1} f(x,y)dxdy$ and $\iint\limits_{B_2} f(x,y)dxdy$ both exist, then $\iint\limits_{B_1 \cup B_2} f(x,y)dx = \iint\limits_{B_1} f(x,y)dx + \iint\limits_{B_2} f(x,y)dx$.

331. If $\iint\limits_{B_1} f(x,y)dxdy$ and $\iint\limits_{B_2} f(x,y)dxdy$ both exist and B_1 and B_2 have no common inner points, then $\iint\limits_{B_1 \cup B_2} f(x,y)dx = \iint\limits_{B_1} f(x,y)dx + \iint\limits_{B_2} f(x,y)dx$.

332. If f is integrable on A, then $|f|$ is also integrable on A and $\iint\limits_A |f(x,y)|dx \leq \left|\iint\limits_A f(x,y)dx\right|$.

333. If f is integrable on A, then $|f|$ is also integrable on A and $\left|\iint\limits_A f(x,y)dx\right| \leq \iint\limits_A |f(x,y)|dx$.

334. If f is integrable on A, then it is continuous on A.

335. If f is continuous on A, then it is integrable on A.

336. One can find such an integrable function $f : D \to \mathbb{R}$ ($D \subset \mathbb{R}^2$) such that

$$\int_c^d \left(\int_a^b f(x,y)dx \right) dy < \int_a^b \left(\int_c^d f(x,y)dy \right) dx.$$

337. If $f : D \to \mathbb{R}$ ($D \subset \mathbb{R}^2$) is a continuous function on N_x and N_x is a normal domain defined as $N_x = \{(x,y) : a \leq x \leq b, c(x) \leq y \leq d(x)\}$, then

$$\iint\limits_{N_x} f(x,y)dxdy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)dy \right) dx.$$

338. If $f : D \to \mathbb{R}$ ($D \subset \mathbb{R}^2$) is a continuous function on N_x and N_x is a normal domain defined as $N_x = \{(x,y) : a \leq x \leq b, c(x) \leq y \leq d(x)\}$, then

$$\iint\limits_{N_x} f(x,y)dxdy = \int_{c(x)}^{d(x)} \left(\int_a^b f(x,y)dy \right) dx.$$

339. If $f : D \to \mathbb{R}$ ($D \subset \mathbb{R}^2$) is a continuous function on N_x and N_x is a normal domain defined as $N_x = \{(x,y) : a \leq x \leq b, c(x) \leq y \leq d(x)\}$, then

$$\iint\limits_{N_x} f(x,y)dxdy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)dx \right) dy.$$

340. The Cartesian coordinates of a point with polar coordinates $(r, \varphi) = (2, \frac{\pi}{2})$ is $(0, 2)$.

341. The Cartesian coordinates of a point with polar coordinates $(r, \varphi) = (1, \frac{\pi}{2})$ is $(0, 4)$.

342. The Cartesian coordinates of a point with polar coordinates $(r, \varphi) = (2, \frac{\pi}{2})$ is $(2, 0)$.

343. The Cartesian coordinates of a point with polar coordinates $(r, \varphi) = (2, \frac{\pi}{2})$ is $(4, 0)$.

344. The Cartesian coordinates of a point with cylindrical coordinates $(r, \varphi, z) = (2, -\frac{\pi}{2}, 1)$ is $(0, -2, 1)$.

345. The Cartesian coordinates of a point with cylindrical coordinates $(r, \varphi, z) = (2, -\frac{\pi}{2}, 1)$ is $(0, -2, -2)$.

346. The Cartesian coordinates of a point with cylindrical coordinates $(r, \varphi, z) = (2, -\frac{\pi}{2}, 1)$ is $(0, 2, 1)$.

347. The Cartesian coordinates of a point with cylindrical coordinates $(r, \varphi, z) = (2, -\frac{\pi}{2}, 1)$ is $(-2, 0, 1)$.

348. The Cartesian coordinates of a point with cylindrical coordinates $(r, \varphi, z) = (2, -\frac{\pi}{2}, 1)$ is $(2, 0, 1)$.

349. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \frac{\pi}{2})$ is $(0, 2, 0)$.

350. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \frac{\pi}{2})$ is $(2, 0, 0)$.

351. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \frac{\pi}{2})$ is $(0, 0, 2)$.

352. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \pi)$ is $(-2, 0, 0)$.

353. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \pi)$ is $(0, 0, -2)$.

354. The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \pi)$ is $(0, -2, 0)$.