Math G2

Mock Exam

- Make sure that your solutions are detailed enough since in case of mistakes/errors I can grade them better.
- Fill out the box below with capital letters.

Name and Neptun code:

Theoretical questions

Please write your true ('T') or false ('F') answers for the theoretical test into the box at the bottom of this page. You can get +2 points for a correct answer, 0 points for no answer and -1 points for a wrong answer.

1. (Q. 29.) For any $n \times n$ matrices A and B, we have $\det(A + B) = \det(A) \cdot \det(B)$.
2. (Q. 52.) If three vectors are linearly independent, then they are on the same plane.
3. (Q. 72.) For $n \times n$ matrices A and B for which $\det(A) \neq 0$ and $\det(B) \neq 0$ we have $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$.
4. (Q. 165.) The matrix of a linear operator might change if we change the basis of the vector space.
5. (Q. 196.) The series $\sum_{n=1}^{\infty} q^n$ converges, if $q < 1$.
6. (Q. 239.) If for a power series $\sum_{n=1}^{\infty} a_n(x - x_0)^n$ we have $0 < \limsup_{n\to\infty} \sqrt{|a_n|} < \infty$, then the radius of convergence is $R = \frac{1}{\limsup_{n\to\infty} \sqrt{|a_n|}}$.
7. (Q. 271.) The set $x^2 + y^2 \leq 1$ (unit disc with its border) is closed.
8. (Q. 278.) Level curves of a function $f : \mathbb{R}^2 \to \mathbb{R}$ are the curves where $f(c) = x$ for a given value of $c \in \mathbb{R}$.
9. (Q. 292.) The Hessian (or Jacobian) matrix of a function $f : \mathbb{R}^n \to \mathbb{R}$ is always symmetric.
10. (Q. 352) The Cartesian coordinates of a point with spherical coordinates $(r, \varphi, \theta) = (2, \frac{\pi}{2}, \pi)$ is $(-2, 0, 0)$.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

Exercises

1. (Pr. 3. Ex. 1.) Does the following system of equations have a unique solution? If yes, solve it! (8 points)

$$\begin{cases} 2x_1 - x_2 - x_3 = \ 4, \\ 3x_1 + 4x_2 - 2x_3 = 11, \\ 3x_1 - 2x_2 + 4x_3 = 11. \end{cases}$$

2. (Pr. 5. Ex. 23.) Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}.$$

   (a) Prove that it is a convergent series! (4 points)

   (b) How many elements should we add up of this series such that our error is smaller than $\varepsilon = 10^{-2}$? (4 points)

3. (Midterm 2 Ex. 3.) Let us consider function $f : [0, \pi] \to \mathbb{R}$ which is defined as $f(x) = 1$. Define the extension of this function f in a way that its extension is defined for every $x \in \mathbb{R}$ and the Fourier series of the extension is a sine Fourier series. Calculate the coefficients of this series! (8 points)

4. (Pr. 11. Ex. 2.) Search for the maximum or minimum of the function

$$f(x, y) = x^2 - 2x + y^2$$

   inside set A where $A = B \cap C$, in which B is the disc with radius 2 centered at the origin, and C is the union of the first, third and fourth quadrants of the coordinate system. (8 points)

5. (Pr. 12. Ex. 9.) Calculate the integral of the function $f(x, y) = \sin\left(\sqrt{x^2 + y^2}\right)$ on the domain T given by the equations $1 \leq x^2 + y^2 \leq 4$ and $x \geq 0$. (8 points)