Calculus 2 (BMETE92AM37, Stpendium Hungaricum) Topic list for the examination

Tamás Tasnádi

June, 2018

General information

- The examination is a 90 minute long written test.
- Approximately 60% of the examination consists of numerical problems similar to the ones in the tests during the semester. The candidates are required to use the methods learned in order to solve the problems.
- Approximately 40% of the examination contains theory. Here the candidates are required to give the definitions of concepts, state theorems, present the proof of some theorems learned in class or prove new theorems with methods similar to the ones learned in class.
- If the result of the exam is less then 40% then the grade is "failed" (1), and the exam has to be repeated.
- Otherwise the overall result R (in percents) is calculated as

  $R = 0.2S + 0.2M_1 + 0.2M_2 + 0.4E$,

  where S, M_1, M_2 and E are the cumulative result of the small tests, the results of first and second midterm tests and the exam, respectively. (All in percents.) The result R is converted to grade according to the table on the right.

result	grade
$0\% \le R < 40\%$	failed (1)
$40\% \le R < 55\%$	sufficient (2)
$55\% \le R < 65\%$	satisfactory (3)
$65\% \le R < 80\%$	good (4)
$80\% \le R \le 100\%$	excellent (5)

- The examination can be repeated with the aim of improving the result. In this case it is not compulsory to submit the exam, however, if it is submitted then it overwrites the previous result so worsening (down to 40%) is also possible.
- The topic list below gives the basic concepts and theorems required. The numbers refer to the book:

  Miklós Laczkovich, Vera, T. Sós: *Real Analysis II. (Series, Functions of Several Variables and Applications)*, 2017, Springer

- **The theorems typeset in boldface should be known with their proofs.** The other theorems should be known and can be used without proofs in solving other problems.

1 Functions from $\mathbb{R}^p$ to $\mathbb{R}$

Operations in $\mathbb{R}^p$, inner product, norm, distance, triangle inequality, Schwarz inequality.

Graph of $\mathbb{R}^p \to \mathbb{R}$ functions.

Convergence of a sequence in $\mathbb{R}^p$. **Convergence is equivalent to coordinate-wise convergence (Theorem 1.5), operations with convergent sequences (Theorems 1.6, 1.7), Cauchy's criterion (Theorem 1.8), Bolzano–Weierstrass theorem (1.9).**

Basic topological concepts (interior, boundary, exterior point; limit point, derived set, isolated point; open, closed set; closure of a set). **Properties of open and closed sets (Theorems 1.13, 1.14, 1.17, 1.18, 1.24). Cantor's theorem (1.25).** Lindelöf's theorem (1.26). Compact set. **Borel's theorem (1.31).**

Limit of a multivariable function. Transference principle (Theorem 1.38). Squeeze theorem (1.39), operations with limits (Theorem 1.40).

Continuity of functions, transference principle (Theorem 1.44), operations with continuous functions (Theorems1.46, **1.48**, 1.49), **Weierstrass's theorem (1.51)**, uniform continuity, **Heine's theorem (1.53)**.

Partial derivatives, local extrema and **their connection (Theorem 1.60)**. Extremal values on a compact set (Theorem 1.61).

Differentiability, (total) derivative (gradient) of a function, its connection to continuity and to the partial derivatives (**Theorems 1.66, 1.67**, Theorem 1.71). Tangent plane of a function. Directional derivative and **its connection to the gradient (Theorem 1.77)**.

Second order partial derivatives, Young's theorem (1.82). Twice differentiability, second order Taylor expansion. Quadratic forms, conditions for positive/negative (semi) definitness. Conditions for local extrema (Theorem 1.101).

2 Functions from $\mathbb{R}^p$ to $\mathbb{R}^q$

Limit and continuity of vector valued multivariable functions. Transference principle (Theorems 2.2, 2.4, 2.5). **Continuous image of compact set is compact (Theorem 2.8).**

Differentiability, **relation to differentiability of components (Theorem 2.13)**. Jacobian matrix. Relation of differentiability to continuity and partial differentiability (Theorem 2.16).

Differentiation rules (Theorem 2.19), chain rule (Theorem 2.20, Corollary 2.23), differential of the inverse function (Theorem 2.26).

Inverse function theorem (2.38), imlicit function theorem (2.40). Lagrange multiplier method (Theorem 2.44).

3 The Jordan measure

Outer, inner measure, null set, Jordan measurability, Jordan measure. Box-counting measures, their limit (Theorem 3.4). **Properties of the inner, outer measure (Theorems 3.6, 3.7, 3.9). Properties of the Jordan measure (Theorems 3.16, 3.17).** Characterization of the Jordan measure by its properties (Theorem 3.18).

Volume of a parallelepiped (Theorem 3.31).

Change of measure under linear transformation (Theorems 3.35, 3.36).

4 Integrals of multivariable functions I

Defining integral on rectangles: partition of a rectangle, lower and upper sums, **their properties (Lemmas 4.2, 4.3)**. Lower, upper integrals; integrability and Riemann integral of a function. Oscillatory sum, approximating sum.

Defining integral on bounded, measurable sets: partitions; lower, upper sums, lower, upper integrals, integrability, integral. Connection to Jordan measure (Theorem 4.11). Integrability of continuous functions (Theorem 4.14).

Calculating the integrals on rectangles (Theorems 4.17, 4.19) and on normal domains (Theorem 4.18) by successive integration. Integration by substitution (Theorem 4.22). Planar polar coordinates, **substitution by polar coordinates (Theorem 4.25)**. Cylindrical coordinates, spherical polar coordinates, **and their Jacibi determinants**. Improper integrals.

5 Integrals of multivariable functions II

Line integral as Stieltjes integral. Calculation of line integrals over differentiable curve (Theorems 5.5, 5.8). Primitive function, Newton–Leibniz formula for line integrals (Theorem 5.11) and **its proof for differentiable curves (Remark 5.12)**.

Conditions for the existence of primitive function (Theorem 5.14, **Theorem 5.17**, Theorems 5.22, 5.32).

Simple closed curve, its direction. Green's theorem (5.34).

Parametrized surface, surface area. Surface area of a graph of a function (Theorem 5.44).

Divergence, rotation of a vector field. Gauss–Ostrogradsky theorem, Stoke's theorem (5.48).

6 Series of functions

Pointwise and uniform convergence of series of functions, absolute convergence. **Weierstrass criterion (Theorem 7.27).** Consequences of uniform convergnece (Theorems 7.36, 7.40, 7.42).

Taylor series and power series. Taylor series of trigonometric, hyperbolic, exponential functions (page 250). Analytic function. Radius and domain of convergence of a power series (Theorem 7.49, Couchy–Hadamard formula 7.101). Continuity, termwise differentiability and integrability of a power series (Theorems 7.51, 7.52). Generalized binomial coefficients, binomial series (pages 255–256).

Trigonometric system, **its orthogonality (Lemma 7.72)**. Trigonometric polynomials, trigonometric series, Fourier expansion (Theorem 7.71). Completeness of the trigonometric system (Theorem 7.77).