α VARIÁNS (Részletes)

1, a, $\ln n \ll n \ll n^{20} \ll 20^n \ll n! \ll n^n$ (4)

b, $\frac{20^n}{n!} = \underbrace{\frac{20}{1}\cdot\frac{20}{2}\cdot\frac{20}{3}\cdot\ldots\cdot\frac{20}{20}}_{K\in\mathbb{R}}\cdot\underset{1}{\underset{\wedge}{\frac{20}{21}}}\cdot\underset{1}{\underset{\wedge}{\ldots}}\cdot\frac{20}{n} \le K\cdot\frac{20}{n}\xrightarrow{n\to\infty} 0$

(7) Tehát $n! \gg 20^n$.

2, Ha f és g deriválható x-ben, akkor $\exists\,(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ (3)

($(fg)' = f'g + fg'$ -re is 3 pontot adunk.)

[8]

$$(fg)'(x) = \lim_{h\to 0}\frac{(fg)(x+h) - (fg)(x)}{h} \ (2) =$$

$$= \lim_{h\to 0}\left(\frac{f(x+h)(g(x+h) - g(x))}{h} + \frac{(f(x+h) - f(x))\,g(x)}{h}\right) \ (3) =$$

$$= \underbrace{\lim_{h\to 0} f(x+h)}\cdot g'(x) + f'(x)\,g(x) = f(x)g'(x) + f'(x)g(x) \ (2)$$

f folyt. x-ben, mert diff.-ható (1)

3, a, [8]

$$\frac{x+15}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3} \ (2) \Longrightarrow x+15 = A(x-3) + B(x+3)$$

$x=+3$: $18 = 6B \Rightarrow \underline{B=3}$

$x=-3$: $12 = -6A \Rightarrow \underline{A=-2}$ (4)

$$\int\frac{x+15}{x^2-9}\,dx = -2\int\frac{1}{x+3}\,dx + 3\int\frac{1}{x-3}\,dx = \underline{\underline{-2\ln|x+3| + 3\ln|x-3| + C}} \ (2)$$

b, [8]

$$\int\frac{x+15}{\sqrt{x^2+9}}\,dx = \frac{1}{2}\int 2x\cdot(x^2+9)^{-1/2}\,dx + 5\int\frac{dx}{\sqrt{1+\left(\frac{x}{3}\right)^2}} \ (2) =$$

$$= \underline{\underline{\cancel{\frac{1}{2}}\cdot(x^2+9)^{1/2}\cdot\cancel{2} \ (3) + 5\cdot\operatorname{arsh}\left(\frac{x}{3}\right)\cdot 3 \ (3) + C}}$$

4, a, $y'(x) + f(x)\,y(x) = q(x)$ ③

[5] b, $\varphi_1 - \varphi_2$ megoldása a homogén egyenletnek. ②

$$\left.\begin{aligned} &\varphi_1' + f\varphi_1 = q \\ \ominus\ &\varphi_2' + f\varphi_2 = q \\ \hline &(\varphi_1-\varphi_2)' + f(\varphi_1-\varphi_2) = 0 \ \checkmark \end{aligned}\right\} ③$$

[8] c, $y' + xy = 2x$ az egyenlet inhomogén lineáris, de egyben szeparálható is, egyszerűbb szeparálhatóként kezelni:

$\dfrac{dy}{dx} = x\,(2-y)$ ② $\Longrightarrow$ $\displaystyle\int \frac{dy}{2-y} = \int x\,dx$

$y \equiv 2$ megoldás $\qquad -\ln|2-y| = \dfrac{x^2}{2} + C$ ② ②

$2 - y = K \cdot e^{-\frac{x^2}{2}} \Longrightarrow \underline{\underline{y_{ált}(x) = \tilde{K}\,e^{-\frac{x^2}{2}} + 2\ ;\ \tilde{K} \in \mathbb{R}}}$ ②

Inhomogén lineárisként kezelve: $y_{H,ált}(x) = K\,e^{-\frac{x^2}{2}}$ ④

$y_{I,part}(x) = 2$ ③ ; $y_{ált}(x) = K\,e^{-x^2/2} + 2$ ①

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5, a, [3] $\displaystyle ch\,x = \sum_{n=0}^{\infty} \frac{1}{(2n)!}\,x^{2n}$ ② ; $K.T. = \mathbb{R}$ ①

b, [8] $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{(2n)!}\cdot x^{2n+3} = x^3 \cdot \sum_{n=0}^{\infty} \frac{(\sqrt{3}\,x)^{2n}}{(2n)!} = \underline{\underline{x^3\,ch(\sqrt{3}\,x)}}$ ②

③ ③

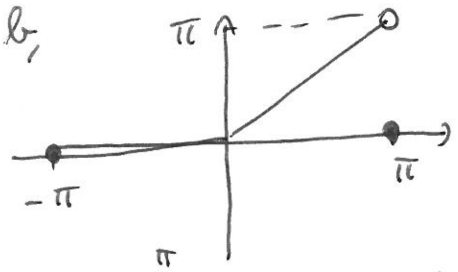
6, Síkbeli polárban: (4) (1)

[10] $$I=\int_{r=0}^{R}\int_{\varphi=0}^{2\pi}(r\cos\varphi)^2\, r\, d\varphi\, dr=\left(\int_{r=0}^{R} r^3\, dr\right)\left(\int_{\varphi=0}^{2\pi}\cos^2\varphi\, d\varphi\right)=$$

$$=\underbrace{\frac{R^4}{4}}_{(2)}\cdot\underbrace{\int_{\varphi=0}^{2\pi}\frac{1+\cos(2\varphi)}{2}\,d\varphi}_{(2)}=\frac{R^4}{4}\left[\frac{\varphi}{2}+\frac{\sin(2\varphi)}{4}\right]_0^{2\pi}=\underline{\underline{\frac{R^4\cdot\pi}{4}}}\quad (1)$$

7,* a, Ha f 2π-szerint periodikus, és $\exists\, x_0=0<x_1<x_2<\dots<x_n=2\pi$, hogy f monoton $\forall\,[x_i,x_{i+1}]$ intervallumon, és $\exists\, f(x_i+0)\in\mathbb{R}$, $\exists\, f(x_i-0)\in\mathbb{R}$ $(i=0,1,2,\dots n-1)$, akkor $\forall x\in\mathbb{R}$ esetén a Fourier-sor ϕ összegére $\phi(x)=\dfrac{f(x+0)+f(x-0)}{2}$ teljesül. (3)

[8] b,



$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\,dx=\frac{1}{\pi}\int_0^{\pi}x\,dx=\frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}=\underline{\underline{\frac{\pi}{2}}}\quad (3)$$

[5] $$b_5=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin(5x)\,dx=\frac{1}{\pi}\int_0^{\pi}x\sin(5x)\,dx \overset{(2)}{=}\frac{1}{\pi}\left[x\,\frac{-\cos(5x)}{5}\right]_0^{\pi}+\frac{1}{\pi}\int_0^{\pi}\frac{\cos(5x)}{5}\,dx=$$

$$=\frac{1}{5}+\frac{1}{25\pi}\left[\sin(5x)\right]_0^{\pi}=\underline{\underline{\frac{1}{5}}}\quad (3)$$

c, $$\phi(\pi)=\frac{f(\pi+0)+f(\pi-0)}{2}=\underline{\underline{\frac{\pi}{2}}}\quad (2)$$

8,* $$\mathcal{F}\left[e^{-|x|}\right](\omega)=\frac{2}{1+\omega^2}$$

$$x=\frac{y+3}{2};\ dx=\frac{1}{2}dy$$

[6] a, $$\mathcal{F}\left[e^{-|2x-3|}\right](\omega)=\int_{-\infty}^{\infty}e^{-i\omega x}e^{-|\overbrace{2x-3}^{y}|}\,dx=\int_{-\infty}^{\infty}e^{-i\omega\frac{y+3}{2}}e^{-|y|}\frac{1}{2}\,dy=$$

$$=\underline{\underline{\frac{1}{2}\cdot e^{-\frac{3i\omega}{2}}\cdot\frac{2}{1+\left(\frac{\omega}{2}\right)^2}}}\quad (3)$$

[6] b, $$\mathcal{F}\left[x e^{-|x|}\right](\omega)=i\cdot\left(\mathcal{F}\left[e^{-|x|}\right](\omega)\right)'=i\cdot\left(\frac{2}{1+\omega^2}\right)'=\underline{\underline{-\frac{2i\cdot 2\omega}{(1+\omega^2)^2}}}\quad (3)$$

B VARIÁNS (Tanár)

1, α - hoz hasonló.

2, Mint C.

3, a, $$\int \frac{x+18}{\sqrt{x^2+4}}\,dx = \frac{1}{2}\int \overbrace{2x\,(x^2+4)^{-1/2}}^{f'\cdot f^{-1/2}\text{ alak}}\,dx + \frac{18}{2}\int \frac{dx}{\sqrt{1+\left(\frac{x}{2}\right)^2}} =$$

$$= \frac{1}{2}\cdot (x^2+4)^{1/2}\cdot 2 + 9\cdot \operatorname{arsh}\left(\frac{x}{2}\right)\cdot 2 + C$$

b, $$\int \frac{x+18}{x^2-4}\,dx = \int\left(\frac{-4}{x+2}+\frac{5}{x-2}\right)dx = -4\ln|x+2| + 5\ln|x-2| + C$$

4, a, b, Mint C,

c, $$\frac{dy}{dx} = x\,(y+3) \Longrightarrow \int \frac{dy}{y+3} = \int x\,dx \Longrightarrow \ln|y+3| = \frac{x^2}{2} + C$$

$$y+3 = A\,e^{\frac{x^2}{2}} \quad ; \quad y_{\text{ált}}(x) = A\,e^{\frac{x^2}{2}} - 3 \quad ; A\in\mathbb{R}$$

5, a, $$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\,x^{2n} \quad ; \quad \forall x\in\mathbb{R}$$

b, $$\sum_{n=0}^{\infty} \frac{(-5)^n}{(2n)!}\cdot x^{2n+5} = x^5 \sum_{n=0}^{\infty} \frac{(-1)^n(\sqrt{5}\,x)^{2n}}{(2n)!} = x^5\cos(\sqrt{5}\,x)$$

6, Mint C,