1.

$$\int \sqrt{x+2}\,dx = \int (u^2-2)^2 \cdot u \cdot 2u\,du = \int (u^4-4u^2+4)\,2u^2\,du$$

u^2-2

$$\int 2u^6-8u^4+8u^2\,du =$$

$2u^4-4u^2$

$$\frac{2u^5}{5}+\frac{4u^3}{3} =$$

$$\frac{2(x+2)^5}{5}-\frac{4(x+2)^3}{3}$$

$u=\sqrt{x+2} \rightarrow x+2=u^2$

$x=u^2-2$

$\frac{du}{dx}=\frac{1}{2}\cdot\frac{1}{\sqrt{x+2}}$

$2\underbrace{\sqrt{x+2}}_{u}\,du=dx$

2.

$$\int_0^1 \frac{x^2+x-3}{x^2-x-2}\,dx = \int_0^1 1-\frac{2x-1}{x^2-x-2}\,dx = \int_0^1 1-\frac{1}{(x+1)(x-2)}\,dx =$$

$x^2-x-2+2x-1$

f / f

$$[x]_0^1 - \int_0^1 \frac{A}{x+1}+\frac{B}{x-2}\,dx = 1-\int_0^1 \frac{-1/3}{x+1}+\frac{1/3}{x-2}\,dx =$$

$1-0$

$A(x-2)+B(x+1)=1$

$A=-B$

$-2A+B=1$

$B=1+2A$

$-A=1+2A$

$-3A=1$

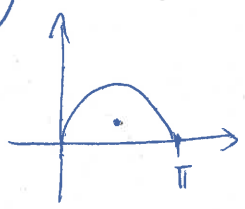
$A=-\frac{1}{3}$

$$1+\frac{1}{3}\left[\ln|x+1|\right]_0^1-\frac{1}{3}\left[\ln|x-2|\right]_0^1 = 1+\frac{2}{3}\ln 2$

$\ln 2$ / $-\ln 2$

$$= \underbrace{[x]_0^1}_{1} - \underbrace{\left[\ln|x^2-x-2|\right]_0^1}_{\ln(2)-\ln(2)} = 1$$

3.



$$x_s = \frac{\int_0^\pi x\sin x\,dx}{\int_0^\pi \sin x\,dx} = \frac{\pi}{2}$$

$$y_s = \frac{\frac{1}{2}\int_0^\pi \sin^2 x\,dx}{\int_0^\pi \sin x\,dx} = \frac{\pi/4}{2} = \frac{\pi}{8}$$

$$\int_0^\pi \sin x\,dx = \left[-\cos x\right]_0^\pi = -(-1)-(-1) = 2$$

$$\int_0^\pi x\sin x\,dx = \underbrace{\left[-x\cos x\right]_0^\pi}_{-\pi(-1)-(-0)\cdot 1=\pi} - \int_0^\pi 1\cdot(-\cos x)\,dx = \pi + \int_0^\pi \cos x\,dx = \pi + \underbrace{\left[\sin x\right]_0^\pi}_{0-0} = \pi$$

f g'

$\hookrightarrow g=(-\cos x)$

$$\frac{1}{2}\int_0^\pi \sin^2 x\,dx = \frac{1}{2}\int_0^\pi \frac{1}{2}-\frac{\cos 2x}{2}\,dx = \frac{1}{4}\int_0^\pi 1\,dx - \frac{1}{8}\int_0^\pi 2\cos 2x\,dx = \frac{\pi}{4}-\frac{1}{8}\underbrace{\left[\sin 2x\right]_0^\pi}_{0-0} = \frac{\pi}{4}$$

4. a) $B^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \frac{1}{\det B}\, adj(B) = \frac{1}{2} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}^T = \frac{1}{2} \cdot \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0{,}5 & -0{,}5 \\ 0 & 1 \end{bmatrix}$

b) $A \cdot C = (3\ -1\ 1) \begin{pmatrix} 2 & -4 \\ 1 & 1 \\ 5 & 0 \end{pmatrix} = (10\ \ -13)$

$C \cdot B = \begin{pmatrix} 2 & -4 \\ 1 & 1 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 2 & 2 \\ 10 & 5 \end{pmatrix}$

$B \cdot B = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}$

5. $P\ (5, -4, 4)$

$x - 2y + 2z = 3$

$1(x-3) - 2(y-0) + 2(z-0) = 0$

$\underline{n} = (1, -2, 2)$

b) $d = \frac{1(5-3) + (-2)(-4-0) + 2(4-0)}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{2 + 8 + 8}{3} = 6$

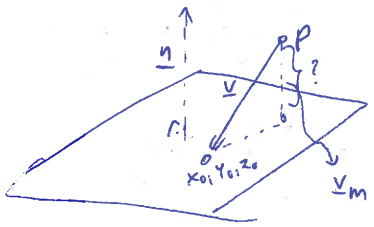
$\sqrt{36} = 6$

$\sqrt{(-2)^2 + 4^2 + (-4)^2}$

hozza

$(-2, 4, -4)$

a)



$\underline{v}_p = \frac{\langle \underline{v}, \underline{n} \rangle}{\|\underline{n}\|^2}\, \underline{n} = (1, -2, 2) \cdot \frac{(0, 5, -4) \cdot \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}{\sqrt{1^2 + (-2)^2 + 2^2}^2} = \frac{0 - 10 - 8}{9} = -2$

$\underline{v} = (x_0 - 5, y_0 + 4, z_0 - 4) = (0, 5, -4)$

$\downarrow$ példa

$(5, 1, 0)$

6. $f(x) = 3 \arcsin(2x - 1) + 1$

$f'(x) = 3 \cdot \frac{1}{\sqrt{1 - (2x-1)^2}} \cdot 2 = \frac{6}{\sqrt{4x - 4x^2}}$

$1 - 4x^2 - 1 + 4x$

$x_0 = \frac{1}{2}$

$y_0 = f\left(\frac{1}{2}\right) = 3 \arcsin\left(2 \cdot \frac{1}{2} - 1\right) + 1 = 1$

$\arcsin 0 = 0$

$f'\left(\frac{1}{2}\right) = \frac{6}{\sqrt{\frac{4}{2} - \frac{4}{2^2}}} = 6$

$2 - 1$

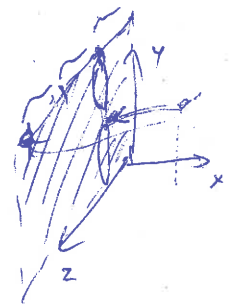
$6 = \frac{y - 1}{x - \frac{1}{2}}$

7.

a)
$$\begin{pmatrix}1\\0\\0\end{pmatrix} \to \begin{pmatrix}a_1\\b_1\\c_1\end{pmatrix} \qquad \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix} \qquad \begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}a_3\\b_3\\c_3\end{pmatrix}$$

$$\begin{pmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{pmatrix} \cdot \begin{pmatrix}x\\y\\z\end{pmatrix} = \square$$

b)



$$\begin{pmatrix}1\\0\\0\end{pmatrix} \to \begin{pmatrix}0\\0\\0\end{pmatrix} \qquad \begin{pmatrix}0\\1\\0\end{pmatrix} \to \begin{pmatrix}0\\2\\0\end{pmatrix} \qquad \begin{pmatrix}0\\0\\1\end{pmatrix} \to \begin{pmatrix}0\\0\\3\end{pmatrix}$$

$$\begin{pmatrix}0&0&0\\0&2&0\\0&0&3\end{pmatrix}\begin{pmatrix}5\\-4\\2\end{pmatrix} = \begin{pmatrix}0\\-8\\6\end{pmatrix}$$

8.

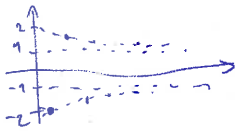
a) NEM ; pl.: $\sin(n)$; $(-1)^n$

b)
$$a_n = \frac{1}{2n+1} \to 0 \qquad a_0 = 1 \to \text{első 1-es.}$$
0 → akár

$$b_n = \frac{n-n^2}{n+1} = \frac{n(1-n)}{n+1} = \frac{\frac{1}{n}-1}{\frac{1}{n}+\frac{1}{n^2}} \to -\infty$$

$$c_n = (-1)^n \frac{n+1}{n} = (-1)^n + \underbrace{\frac{1}{n}(-1)^n}_{\to 0}$$

$a_1 = (-1) + (-1) = (-2)$



g)

a)
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} \overset{\infty}{\underset{\infty}{}} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)} = \ldots = \lim_{x\to\infty} \frac{f^{(n)}(x)}{g^{(n)}(x)}$$

$\frac{\infty}{\infty}$; $\frac{0}{0}$; ...

ha ez létezik

b)
$$\lim_{x\to\infty} \frac{2x+1}{\ln(2x-1)} = \lim_{x\to\infty} \frac{2}{\frac{1}{2x-1}\cdot 2} = \lim_{x\to\infty} 2x-1 = \infty$$

$$\frac{\overbrace{n^2+1+2n}^{-n-n^2}}{(n+1)-(n+1)^2} - \frac{n-n^2}{n+1} = \frac{n+1-n^2-1-2n}{n+2} - \frac{n-n^2}{n+1}$$