1.

$$\int \frac{2x}{x^2-4x+8}\,dx = \int \underbrace{\frac{2x-4}{x^2-4x+8}}_{f'/f}\,dx + \int \frac{4}{x^2-4x+8}\,dx = \%$$

2 pont

$$\frac{4}{x^2-4x+8} = \frac{1}{\left(\frac{x-2}{2}\right)^2+1}$$

$$\% = \ln\left|x^2-4x+8\right| + 2\cdot \arctan\left(\frac{x-2}{2}\right) + c$$

2 pont ↑ 3 pont ↑

2.

$$\int_0^1 \left(\frac{x\sqrt[4]{x}}{\sqrt{x}}\right)^{\frac{2}{3}} + \frac{3x^2}{\sqrt[3]{x^3+1}}\,dx = \int_0^1 \sqrt{x}\,dx + \int_0^1 (3x^2)\cdot(x^3+1)^{-\frac{1}{3}}\,dx = \%$$

2 pont, 3 pont

$$\left(\frac{x^{\frac{5}{4}}}{x^{\frac{1}{2}}}\right)^{\frac{2}{3}} = \left(x^{\frac{3}{4}}\right)^{\frac{2}{3}} = x^{\frac{1}{2}} = \sqrt{x}$$

$$3x^2\cdot(x^3+1)^{-\frac{1}{3}} = f'\cdot(f)^{-\frac{1}{3}}$$

$$\% = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + \left[\frac{(x^3+1)^{\frac{2}{3}}}{\frac{2}{3}}\right]_0^1 = \left(\frac{1}{\frac{3}{2}} - 0\right) + \left(\frac{2^{\frac{2}{3}}}{\frac{2}{3}} - \frac{1}{\frac{2}{3}}\right) = \frac{2}{3} + \frac{\sqrt[3]{4}}{2/3} - \frac{3}{2} \approx 1{,}548$$

1 pont ↑ 1 pont ↑

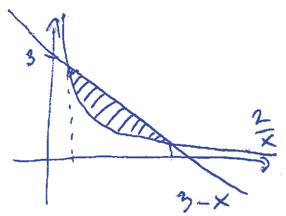
3.

$xy = 2$

$y = \frac{2}{x}$

$x + y = 3$

$y = 3 - x$



metszés: → 2 pont

$\frac{2}{x} = 3 - x$

$0 = -x^2+3x-2 = -(x-1)(x-2)$ → $x_1 = 1$, $x_2 = 2$

$$\int_1^2 3 - x - \frac{2}{x}\,dx = \quad \leftarrow \text{2 pont}$$

$$\left[3x - \frac{x^2}{2} - 2\ln|x|\right]_1^2 =$$

$$(6 - 2 - 2\ln 2) - (3 - \tfrac{1}{2} - 0) =$$

3 pont → $1{,}5 - 2\ln 2$

4.

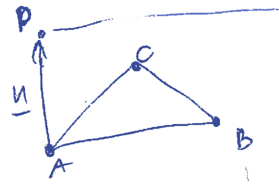
$\vec{AB}\ (-5, 6, 2)$

$\vec{AC}\ (1, -2, -2)$

A (3, −1, 4)
B (−2, 5, 6)
C (4, −3, 2)

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -5 & 6 & 2 \\ 1 & -2 & -2 \end{vmatrix} = (-8, -8, 4) = \underline{n}$$

normálvektor



$(3 - 8t;\ -1 - 8t;\ 4 + 4t)$ ← P pont paraméteresen

$$|\vec{AP}| = \sqrt{(-8t)^2 + (-8t)^2 + (4t)^2} = 12t = 12$$

64 64 16

$\Rightarrow t = 1$

$\Rightarrow P = (3 - 8\cdot 1,\ -1 - 8\cdot 1,\ 4 + 4\cdot 1)$

$(-5,\ -9,\ 8)$

sík egyenlete:

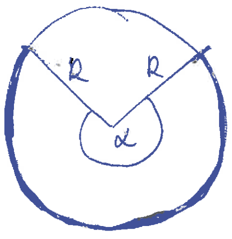
$(-8)\cdot(x-3) + (-8)(y+3) + 4(z-2) = 0$

5.

$$\left(\frac{n+2}{n-1}\right)^{3-2n} = \left(1 + \frac{3}{n-1}\right)^{3-2n} = \underbrace{\left(1 + \frac{3}{n-1}\right)^{1}}_{\downarrow\ 1} \cdot \underbrace{\left(1 + \frac{3}{n-1}\right)^{-2(n-1)}}_{\downarrow\ e^{3\cdot(-2)}} = e^{-6}$$

$$\left(\lim_{n\to\infty}\left(1 + \frac{k}{n}\right)^{m\cdot n} \to e^{k\cdot m}\right)$$

6.

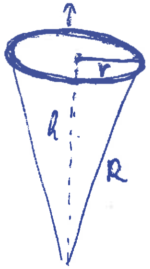


$$2R\cdot\pi\cdot\frac{\alpha}{360^\circ}$$

$$\beta := \frac{\alpha}{360^\circ} \qquad 2R\cdot\pi\cdot\beta$$

$$\| \quad \rightarrow r = R\cdot\beta$$

$$2\cdot r\cdot\pi$$



$$V = \frac{1}{3}\cdot r^2\cdot\pi\cdot h =$$
$$\frac{1}{3}\cdot r^2\cdot\pi\sqrt{R^2-r^2} =$$
$$\frac{1}{3}(R\cdot\beta)^2\pi\cdot\sqrt{R^2-(R\cdot\beta)^2} =$$
$$\frac{1}{3}R^2\cdot\beta^2\cdot\pi\cdot R\sqrt{1-\beta^2} =$$
$$\frac{1}{3}R^3\cdot\pi\cdot\sqrt{\beta^4-\beta^6}$$

$$\frac{d}{d\beta}V = \frac{R^3\cdot\pi}{3}\cdot\frac{1}{2}\underbrace{(\beta^4-\beta^6)^{-\frac{1}{2}}}\cdot\underbrace{(4\beta^3-6\beta^5)} = 0$$

$R\neq 0$

$\beta^4-\beta^6\neq 0$

$\Rightarrow \beta\neq 0$
$\beta\neq 1$
$\beta\neq(-1)$

$\hookrightarrow 4\beta^3-6\beta^5=0$

$2\beta^3(2-3\beta^2)=0$

$2-3\beta^2=0$

$\beta=\sqrt{\frac{2}{3}}$

$\Rightarrow \alpha=\sqrt{\frac{2}{3}}\cdot 360^\circ\approx 294^\circ$

7.

a) $\det\begin{pmatrix}1&1\\2&3\end{pmatrix}=3-2=1$ $\Rightarrow$ egyértelmű megoldás

$\det\begin{pmatrix}1&1\\2&2\end{pmatrix}=0$ $\Rightarrow$ nincs egyértelmű megoldás, ha egyáltalán van

b)

$\left(\begin{array}{cc|c}1&1&2\\2&2&3\end{array}\right)\rightarrow\left(\begin{array}{cc|c}1&1&2\\0&0&-1\end{array}\right)$ $\rightarrow$ nincs megoldás („ellentmondó" sor)

$\left(\begin{array}{cc|c}1&1&2\\2&2&4\end{array}\right)\rightarrow\left(\begin{array}{cc|c}1&1&2\\0&0&0\end{array}\right)$ $\rightarrow$ végtelen sok megoldás

8.

$f(x)$ $[a,b]$-n folyt. (a,b)-n diffható és $f(a)=f(b)$

$\Rightarrow \exists\, x_0\in(a,b)$, hogy $f'(x_0)=0$

$f(x)=x \Rightarrow$ ✗ $\quad f(-1)=(-1)\neq 1=f(1)$

$g(x)=|x| \Rightarrow$ ✗ $\quad$ nem diffható 0-ban (pedig $0\in(-1,1)$)

$h(x)=x^2 \Rightarrow$ ✓ $\quad h'(x)=2x=0\rightarrow x_0=0$

$j(x)=\frac{1}{x} \Rightarrow$ ✗ $\quad$ nem folyt. 0-ban (pedig $0\in[-1,1]$)

$\lim_{x\to 0+}\frac{1}{x}=\infty \qquad \lim_{x\to 0-}\frac{1}{x}=-\infty$

9.

$$\int_{1/2}^{1} \frac{1}{\sqrt{2x-1}}\,dx = \lim_{\varepsilon \to 0} \int_{1/2+\varepsilon}^{1} \frac{1}{\sqrt{2x-1}}\,dx =$$

f' $(f)^{-\frac{1}{2}}$

$$\left[\int \left(\sqrt{2x-1}\right)^{-1} dx = \frac{1}{2}\int 2(2x-1)^{-\frac{1}{2}}\,dx = \cancel{\frac{1}{2}}\,\frac{(2x-1)^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} = \sqrt{2x-1} \right]$$

$$= \lim_{\varepsilon \to 0} \left[\sqrt{2x-1}\right]_{\frac{1}{2}+\varepsilon}^{1} = \lim_{\varepsilon \to 0} \left(1 - \underset{\substack{\downarrow \\ 0}}{\sqrt{\varepsilon}}\right) = 1$$