Asymptotic behaviour of random walks with long memory (summary of PhD thesis)

Bálint Vető

In the present thesis, we consider self-interacting random walks and processes. The central model of our investigations is the *myopic self-avoiding walk* (or 'true' self-avoiding walk). It is a self-repelling random walk, that is, it is pushed to areas which were less visited in the past. More precisely, it is a nearest neighbour random walk on the integer lattice $\mathbb{Z}^d$ which is driven locally by the negative discrete gradient of its own local time. The local time is the number of steps spent on the vertices (or edges) of the underlying lattice.

The myopic self-avoiding walk has been investigated since the early 1980's and it is believed that, in one and two dimensions, the walk behaves super-diffusively (the typical order of the displacement is $t^{2/3}$ in 1D and $t^{1/2}\log^{1/4} t$ in 2D after time t), whereas it is diffusive in three or higher dimensions. In addition, the one-dimensional scaling limit was expected to be a non-standard stochastic process which has been constructed by Tóth and Werner and called the *true self-repelling motion*.

The construction of the true self-repelling motion motivates the first result of the thesis. We consider independent Brownian motions $B(t)$, $X(t)$ and $Y(t)$ in one dimension. Let $X^+(t)$ and $Y^-(t)$ be the trajectories of $X(t)$ and $Y(t)$ pushed upwards and respectively downwards by $B(t)$ according to Skorohod reflection. We show by elementary observations that the distance of the reflected trajectories $X^+(t) - Y^-(t)$ is a three-dimensional Bessel process.

Next, we consider a one-dimensional variant of the myopic self-avoiding walk where the local time is defined on the *oriented edges* of $\mathbb{Z}$. This little change in the definition results in a surprisingly new phenomenon: we prove that the scaling behaviour is different from the other one-dimensional myopic self-avoiding walk models. Instead of the 2/3th power, the proper scaling of the walk turns out to be square root of time. We use a Ray – Knight approach to show that the rescaled local time process of the walk converges to a deterministic triangular shape and to give a local limit theorem for the position of the random walker.

In the *continuous time* version of the original myopic self-avoiding walk, we prove that the right scaling is 2/3th power of time in accordance with former conjectures, and we identify the scaling limit which is the true self-repelling motion. With the Ray – Knight method, we can describe the walk stopped at inverse local times. We give the limit of the rescaled local time profile of the stopped walk in terms of reflected and absorbed Brownian motions and a local limit theorem for the displacement. These are the *first mathematically rigorous results* for a self-repelling random walk model with site repulsion.

Finally, we turn our attention to the *self-repellent Brownian polymer* model which is the *continuous space* counterpart of the myopic self-avoiding walk. It is driven locally by a similar self-repelling mechanism to less visited domains. In three or higher dimensions, we give diffusive upper and lower bounds on the variance of the displacement and *central limit theorem* in terms of the finite dimensional marginal distributions. The main tool is the non-reversible version of the Kipnis – Varadhan type central limit theorem for additive functionals of ergodic Markov processes and the *graded sector condition* of Sethuraman, Varadhan and Yau. These results settle parts of the conjectures about self-repellent random walks and processes.