Tilings of the Aztec diamond on restriced domains

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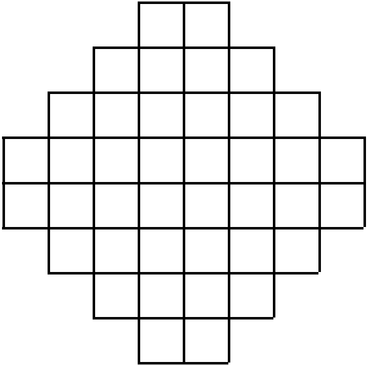
joint work with Patrik L. Ferrari



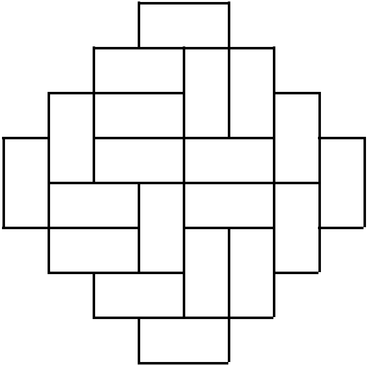
The Aztec diamond domain

Tilings of the Aztec diamond domain: one chosen uniformly from all possible tilings with 1×2 or 2×1 dominos

Introduced as a tiling model by Elkies, Kuperbert, Larsen, Propp in 1992

Tilings of the Aztec diamond

joint work with Patrik L. Ferrari



A sample tiling

We are interested in the boundary fluctuations of the north polar region.

Arctic circle theorem

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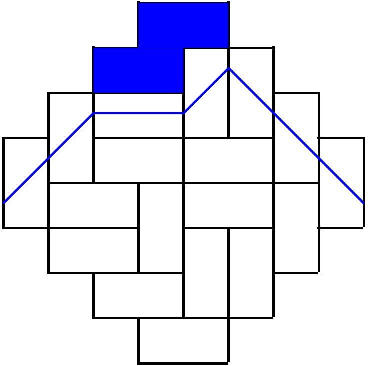

The north polar region

Polar regions: domains with dominos following a completely regular pattern at the corners

Theorem (Jockush, Propp, Shor, 1998, Arctic circle theorem)

The boundary of the polar region converges to a circle as the size of the domain grows to infinity.

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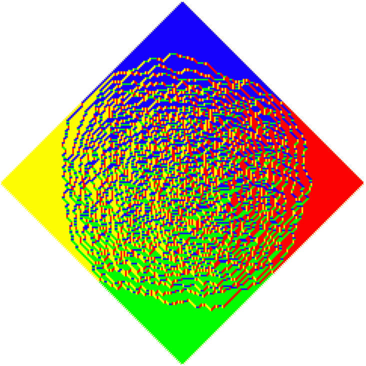


Sample tiling of large size

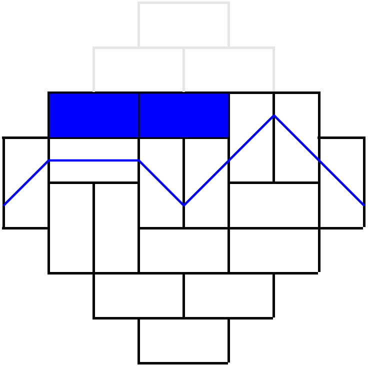
Theorem (Johansson, 2005)

Let $X_n(t)$ denote the boundary of the north polar region in the Aztec diamond of size n. Then

$$\frac{X_n(n^{2/3}t) - c_1 n}{c_2 n^{1/3}} \stackrel{\mathrm{d}}{\Longrightarrow} \mathcal{A}_2(t) - t^2$$

as $n \to \infty$ where $\mathcal{A}_2$ is the Airy$_2$ process.

Tilings of a restricted domain



Sample tiling of the restricted Aztec diamond

Cut off the Aztec diamond with a horizontal line at $c_1 n + Rn^{1/3}$.

Pick a uniform tiling of the remaining domain.

Equivalently, condition the tiling to consist of only horizontal tiles above the line.

Let $X_n^R(t)$ denote the boundary of the north polar region in the restricted model.

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Limit of the boundary of the north polar region

Theorem (P. Ferrari, B. V., 2019)

As $n \to \infty$, we have

$$\frac{X_n^R(n^{2/3}t) - c_1 n}{c_2 n^{1/3}} \longrightarrow \mathcal{A}_2^R(t)$$

where $\mathcal{A}_2^R(t)$ is $\mathcal{A}_2(t) - t^2$ the Airy$_2$ process minus a parabola conditioned on staying below R all the time.

Convergence above is meant in terms of continuum statistics and finite dimensional distributions.

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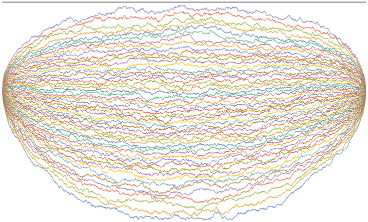
Hard-edge tacnode process



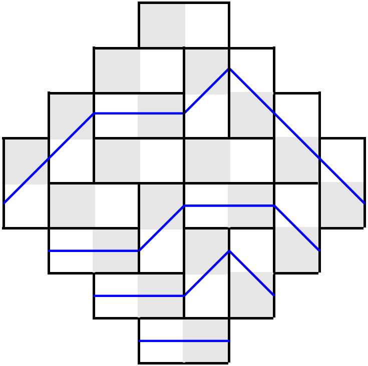
Non-intersecting Brownian paths conditioned to stay below a threshold

The limit process $\mathcal{A}_2^R(t)$ is the top line of the hard-edge tacnode process.

Theorem (P. Ferrari, B. V., 2017)

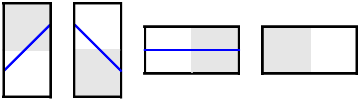
There exists a process (hard-edge tacnode process) characterized by an explicit correlation kernel which arises as the limit of non-intersecting Brownian paths conditioned to stay below a constant level.

Proof ideas



Non-intersecting lines corresponding to a tiling

Rules to map a tiling to a line ensemble:



We represent the line ensemble as non-intersecting random walks.

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The end

Thank you for your attention!